Non-circular cylindrical vessels with quasi-ellipsoidal ends

Krivoshapko S.N. Ivanov V.N,Encyclopedia of Analytical Surfaces ,Springer,2015

https://en.wikipedia.org/wiki/Superellipse

The forming of *quasi-ellipsoidal surfaces* is based on mathematical transformations applied to a canonic equation of *ellipsoid*. V.A. Nikityuk picked up three groups of quasi-ellipsoidal surfaces.

(1) *Quasi-ellipsoidal surfaces with three values of the semi-axes* are given by an equation:

$$\left(\frac{|x|}{a}\right)^n + \left(\frac{|y|}{b}\right)^m + \left(\frac{|z|}{c}\right)^k = 1,$$

where n, m, k are positive numbers.

A quasi-ellipsoid of this type has a closed surface with maximal dimensions along the axes x, y, z equal to $2a$, $2b$, $2c$, accordingly.

(2) *Quasi-ellipsoidal surfaces with six values of the semi-axes* are given by an equation:

$$\left(\frac{|x|}{a_1\theta(-x)+a_2\theta(x)}\right)^n + \left(\frac{|y|}{b_1\theta(-y)+b_2\theta(y)}\right)^m + \left(\frac{|z|}{c_1\theta(-z)+c_2\theta(z)}\right)^k = 1,$$

where n, m, k are positive numbers, $\theta(\xi)$ is *a Heaviside function*; $\theta(\xi) = 0$ if $\xi < 0$ and $\theta(\xi) = 1$ if $\xi \geq 0$. Application of Heaviside function gives an opportunity to introduce six different values of semi-axes of the quasi-ellipsoidal surface: a_1 when $x < 0$ and a_2 when $x \geq 0$; b_1 when $y < 0$ and b_2 when $y \geq 0$; c_1 when $z < 0$ and c_2 when $z \geq 0$. The quasi-ellipsoid has a closed surface with maximum dimensions along the axes x, y, and z equal to sum of the semi-axes: $a_1 + a_2$, $b_1 + b_2$ and $c_1 + c_2$, accordingly. If a quasi-ellipsoidal surface has different values of the semi-axes a_i, b_i, c_i, then it will not be symmetrical relatively to the coordinate planes yOz, xOz and xOy. The values of the exponents of n, m, k define the sign of the curvature of the segments of the surface and the existence of ribs.

(3) *Quasi-ellipsoidal surfaces with cylindrical insertions* along the axis z may be given by an equation:

$$\left(\frac{|x|}{a_1\theta(-x)+a_2\theta(x)}\right)^n + \left(\frac{|y|}{b_1\theta(-y)+b_2\theta(y)}\right)^m + \left(\frac{z\theta(z)+|(z+c_0)\theta(-z-c_0)|}{c_1\theta(-z)+c_2\theta(z)}\right)^k = 1,$$

where n, m, k are positive numbers, $\theta(\xi)$ is a Heaviside function; $\theta(\xi) = 0$ if $\xi < 0$ and $\theta(\xi) = 1$ if $\xi \geq 0$. Application of Heaviside function gives an opportunity to introduce six different values of semi-axes of the quasi-ellipsoidal surface: a_1 when $x < 0$ and a_2 when $x \geq 0$; b_1 when $y < 0$ and b_2 when $y \geq 0$; c_1 when $z < 0$ and c_2 when $z \geq 0$. The quasi-ellipsoid has a closed surface with maximum dimensions along the axes x, y and z equal to sum of the semi-axes: $a_1 + a_2$, $b_1 + b_2$, and $c_1 + c_2 + c_0$, accordingly.

A quasi-ellipsoid of this type may contain a cylindrical insertion by the length c_0 oriented along the axis z. A director line of the cylindrical part oriented along the axis z coincides with the line of the quasi-ellipsoid—the plane xOy intersection.

■ Quasi-ellipsoidal Surface with Three Values of Semi-axes

A quasi-ellipsoidal surface with three values of semi-axes is a closed surface given by an implicit equation:

$$\left(\frac{|x|}{a}\right)^n + \left(\frac{|y|}{b}\right)^m + \left(\frac{|z|}{c}\right)^k = 1,$$

where n, m, k are positive numbers.

A quasi-ellipsoid of this type has maximum dimensions along the axes x, y, z equal to $2a$, $2b$, $2c$, accordingly, where a, b, c are three semi-axes of a quasi-ellipsoid.

In Fig. 1, the quasi-ellipsoidal surface with semi-axes $a = 2$ m, $b = 1$ m, $c = 3$ m and with the values of the degrees $n = m = 2.5$; $k = 0.5$ is shown. The net on the surface is formed by parallels obtained by crossings of the surface by the planes that are perpendicular to the axis Oz, and by

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The vessel consists of a shell with a non-circular cylindrical surface and a quasi-ellipsoidal, smoothly bonded bottoms with a shell.

$\theta(x) := \frac{1}{1 + e^{-100 \cdot x}}$ Approximation of Heaviside function

$a := 1.2 \quad b := 0.6 \quad c := 1 \quad n := 4 \quad d := 1$

$t0 := time(1)$

Dragilev's Method

1.Find the line of intersection of the given surface with the plane Y=0

$$f_1 := \left(\frac{|x_1|}{a}\right)^n + \left(\frac{|x_2|}{b}\right)^n + \left(\frac{x_3 \cdot \theta(x_3) + |(x_3 + d) \cdot \theta(-x_3 - d)|}{c}\right)^n - 1$$

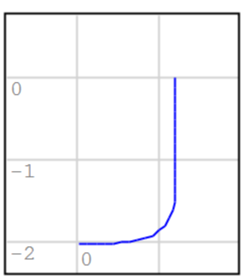
$f_2 := x_2$ уравнение горизонтальной плоскости Z=0

For the starting point of the surface we take a point lying on the axis OY

$X0_1 := a \qquad X0_2 := 10^{-9} \qquad X0_3 := 10^{-9}$

$tmin := 0 \qquad tmax := 3 \qquad \Delta t := 0.1 \qquad N := \frac{tmax}{\Delta t} \qquad N = 30$

$B1 := submatrix(D(X0, tmin, tmax, N), 1, N, 2, 4)$ матрица координат найденной кривой



augment(col(B1, 1), col(B1, 3))

2.Through each point hold the plane x3 = C and find the line of intersection of that plane with a given surface.These lines represent a given surface.

$tmin := 0 \qquad tmax := 6.5 \qquad \Delta t := 0.1 \qquad N1 := \frac{tmax}{\Delta t} \qquad N1 = 65$

Code for multi-colored lines

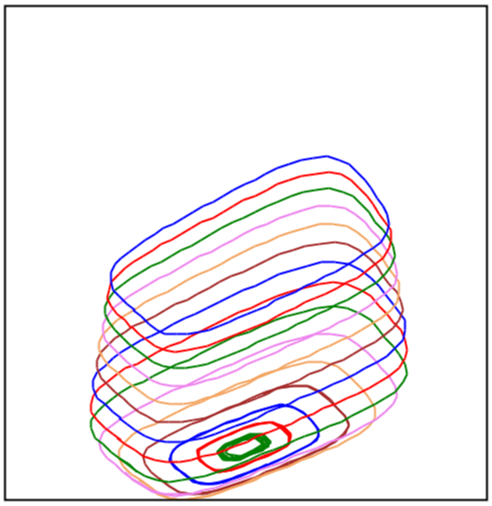
```
for k ∈ 1 , 3 .. N
   (f_2 := x_3) = "Второе уравнение"
   γ := [ -0.8232  -0.4194  0.3827
           0.5677  -0.6187  0.543
           0.009    0.6643  0.7474 ]
   "Находим линии пересечения плоскостей с заданной  поверхностью "
   Lpγ_k := submatrix(D(row(B1 , k)^T , 0 , tmax , N1) , 1 , N1 , 2 , 4)·γ·6
   for i ∈ 1 .. N1
      A_k i := submatrix(Lpγ_k , 1 , i , 1 , 3)
      "Создаем матрицу всех линий сетки"
      if k = 1
         L_i := [A_k i]
      else
         L_i := eval(stack(L_i , [A_k i]))
```

Code for single-color lines

```
for k ∈ 1 , 3 .. N
   (f_2 := x_3) = "Второе уравнение"
   γ := [ -0.8232  -0.4194  0.3827
           0.5677  -0.6187  0.543
           0.009    0.6643  0.7474 ]
   "Находим линии пересечения плоскостей с заданной  поверхностью "
   Lpγ_k := submatrix(D(row(B1 , k)^T , 0 , tmax , N1) , 1 , N1 , 2 , 4)·γ·6
   for i ∈ 1 .. N1
      A_k i := submatrix(Lpγ_k , 1 , i , 1 , 3)
      "К этой матрице добавляем 5 строк кавычек,чтобы  цвет линий повторялся"
      S_k i := [A_k i^T  10^10  10^10  10^10  10^10  10^10]^T
      "Создаем матрицу всех линий сетки"
      if k = 1
         L_i := S_k i
      else
         L_i := stack(L_i , S_k i)
```

$\tau := 1 .. (2 \cdot N1 + 20 - 1)$

Non-circular cylindrical vessel



```
if (t > N1) ∧ (t ≤ N1 + 20)
  | t := N1
else
  if t > N1 + 20
    t := 2·N1 + 20 − t
  else
    t := t
eval(mat2sys_1(L_t))
```

time(1) − t0 = 2.0107 *min*