Find the real roots of

$$f(d) := 0.5\cdot\left(\frac{5.96}{\frac{\pi}{32}\cdot\frac{\left(0.033^4 - d^4\right)}{0.033}}\right) + 0.5\cdot\left[\sqrt{\left(\frac{5.96}{\frac{\pi}{32}\cdot\frac{\left(0.033^4 - d^4\right)}{0.033}}\right)^2 + 4\cdot\left[\frac{61}{\frac{1}{4}\cdot\pi\cdot\left(0.033^2 - d^2\right)}\right]^2}\right] - 400\cdot 10^6$$

Isolating a from

$$0.5\cdot a + 0.5\cdot\sqrt{a^2 + 4\cdot b^2} - c = 0$$

we get

$$ao := -\frac{(b-c)\cdot(b+c)}{c}$$

Identifying

$$a := \frac{5.96}{\frac{\pi}{32}\cdot\frac{\left(0.033^4 - d^4\right)}{0.033}} \qquad b := \frac{61}{\frac{\pi}{4}\cdot\left(0.033^2 - d^2\right)} \qquad c := 400\cdot 10^6$$

the roots of f are in the roots of the polynom P

$$P(d) := \mathrm{numden}(a - ao)_1$$

which is a polynom of degree 8 because

$$\frac{d^9}{dx^9}P(x) = 0$$

for all x in R

Extracting the polynomial coefficients

$$\left|\begin{array}{l} x := 0 \\ \text{for } k \in [0..8] \\ \quad C_{k+1} := \frac{1}{k!}\cdot\frac{d^k}{dx^k}P(x) \end{array}\right. \qquad \frac{C}{10^{30}} = \begin{bmatrix} -0.00000000552883 \\ 0 \\ 0.000010153958008 \\ 0 \\ 0.000019772281339 \\ 0 \\ -8.59839935422905 \\ 0 \\ 3947.84176043574 \end{bmatrix}$$

We can try to solve it with

$$\mathrm{polyroots}(C) = \begin{bmatrix} 0.032817448845286 \\ -0.032817448316602 \\ -0.033073822796661 + 0.000145703373189\cdot i \\ -0.033073822796661 - 0.000145703373189\cdot i \\ 0.033073822531374 + 0.000145695492564\cdot i \\ 0.033073822531374 - 0.000145695492564\cdot i \\ 9.45632461224477\cdot 10^{-13} + 0.032965102242803\cdot i \\ 9.45632461224477\cdot 10^{-13} - 0.032965102242803\cdot i \end{bmatrix}$$

but we can make a better calculus. Notice that all coefficients of odd powers are zero. Thus, this polynom can be symbolically solved as a quartic equation in d^2. Let's try to solve it numerically.

Solving for d^2

$$r2 := \mathrm{polyroots}\left(C_{[1,3..9]}\right) = \begin{bmatrix} 0.001086869787103 \\ 0.001088249362357 \\ 0.001089578855439 \\ -0.001086698004898 \end{bmatrix}$$

we can get some numerical aproach to the roots

$$ro := \overrightarrow{\sqrt{r2}} = \begin{bmatrix} 0.032967708247663 \\ 0.032988624741826 \\ 0.033008769371775 \\ 0.032965102834634\cdot i \end{bmatrix}$$

Lets to find intervals with amplitude 2ε for the roots using above results as guesses $\varepsilon := 10^{-15}$

$d_1 := \text{roots}\left(P(d), d, ro_1\right) = 0.03296485612112$

$f\left(d_1 - \varepsilon\right) = -0.0071$

$f\left(d_1 + \varepsilon\right) = 0.0156$

$d_2 := \text{roots}\left(P(d), d, ro_3\right) = 0.033000246667148$

$f\left(d_2 - \varepsilon\right) = 1.206$

$f\left(d_2 + \varepsilon\right) = -2.0333$

For symmetry $d_3 := -d_1$ $f\left(d_3 - \varepsilon\right) = 0.0156$ $f\left(d_3 + \varepsilon\right) = -0.0071$

$d_4 := -d_2$ $f\left(d_4 - \varepsilon\right) = -2.0333$ $f\left(d_4 + \varepsilon\right) = 1.206$

That is:four real roots, very closed to the discontinuity at d = 0.033. Notice that using do(2) as guess we get it

$$\text{roots}\left(P(d), d, ro_2\right) = \frac{33}{1000}$$

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