```
plot(data, char, size, clr):= | for k∈1..rows(data)
                              |   | r3 k:= char
                              |   | r4 k:= size
                              |   | r5 k:= clr
                              | augment(data, r3, r4, r5)
```

```
appVersion(3)= "0.98.6179"
Created= "jmG, 2018_10_17"
```

NBS fixed points Monograph 125

Helium NBP	-268.935
Hydrogen TP	-259.340
Hydrogen NBP	-252.870
Neon TP	-248.595
Neon NBP	-246.048
Oxygen TP	-218.789
Nitrogen TP	-210.002
Nitrogen NBP	-195.802
Oxygen NBP	-182.962
Carbon Dioxide SP	-78.476
Mercury FP	-38.862
Ice Point	0.00
Ether TP	26.87
Water BP	100.00
Benzoic TP	122.37
Indium FP	156.634
Tin FP	231.9681
Bismuth FP	271.442
Cadmium FP	321.108
Lead FP	327.502
Mercury BP	356.66
Zinc FP	419.580
Sulfur BP	444.674
Cu-Al FP	548.23
Antimony FP	630.74
Aluminum FP	660.37
Silver FP	961.93
Gold FP	1064.43
Copper FP	1084.5

$$T := \begin{bmatrix} -6.2563 & -268.935 \\ -6.2292 & -259.34 \\ -6.1977 & -252.87 \\ -6.1536 & -246.048 \\ -5.873 & -218.789 \\ -5.7533 & -210.002 \\ -5.5356 & -195.802 \\ -5.3147 & -182.962 \\ -2.7407 & -78.476 \\ -1.4349 & -38.862 \\ 0 & 0 \\ 1.0679 & 26.87 \\ 4.2773 & 100 \\ 5.3414 & 122.37 \\ 7.0364 & 156.634 \\ 11.0133 & 231.9681 \\ 13.2188 & 271.442 \\ 16.0953 & 321.108 \\ 16.4733 & 327.502 \\ 18.2179 & 356.66 \end{bmatrix}$$

```
NBP= "NormalBoilingPoint"
TP= "TriplePoint"
SP= "SublimationPoint"
BP= "BoilingPoint"
FP= "FreezingPoint"
```

$$J := \begin{bmatrix} -8.0957 & -210.002 \\ -7.7963 & -195.802 \\ -7.4807 & -182.962 \\ -3.7187 & -78.476 \\ -1.9063 & -38.862 \\ 0 & 0 \\ 1.3739 & 26.87 \\ 5.2677 & 100 \\ 6.4886 & 122.37 \\ 8.3743 & 156.634 \\ 12.5517 & 231.9681 \\ 14.7427 & 271.442 \\ 17.4928 & 321.108 \\ 17.8462 & 327.502 \\ 19.456 & 356.66 \\ 22.9259 & 419.58 \\ 24.3123 & 444.674 \\ 30.1095 & 548.23 \\ 34.9108 & 630.74 \\ 36.6933 & 660.37 \\ 55.669 & 961.93 \\ 61.716 & 1064.43 \\ 62.880 & 1084.5 \end{bmatrix}$$

Part 1: Popular Segments [low cryogenic omitted]

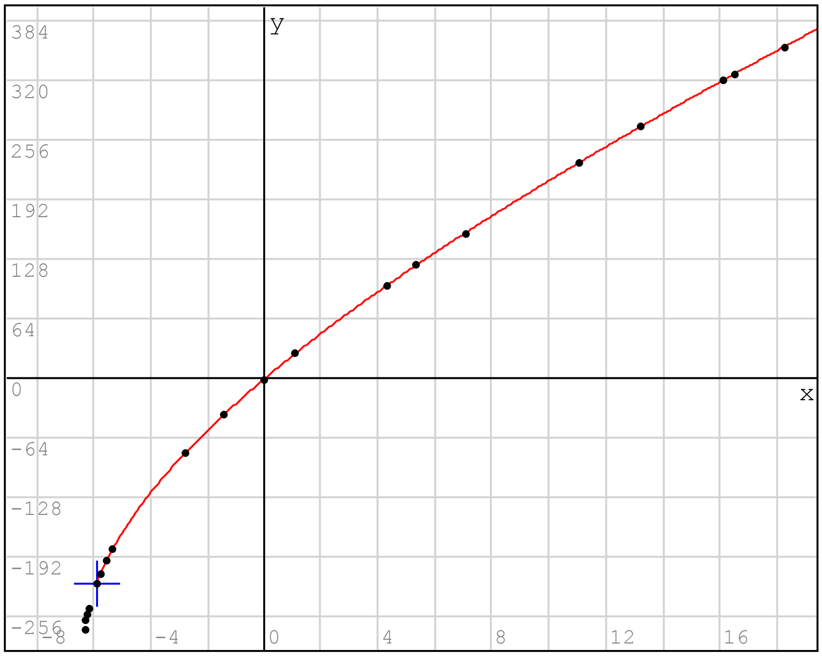
$XY := T_{5..19\ 1..2}$

$$\begin{aligned} &X := \mathrm{col}(XY, 1) \\ &Y := \mathrm{col}(XY, 2) \\ &T := \mathrm{plot}(T, \text{"."}, 12, \text{"black"}) \end{aligned}$$

Popular Segments:
Both fitting functions were done Mathcad 11 [2003].
They were done from MCD GenfitMatrix Padé rational ,
then converted to J-Frac continued fractions.

$$c(x) := \begin{cases} 1 & \text{if } -5.873 \le x \\ \text{""} & \text{otherwise} \end{cases}$$

$$T356(x) := 1579.4 - \cfrac{42563.52}{x + 0.2335992 - \cfrac{1757.435}{x + 104.3645 + \cfrac{11790.76}{x - 69.55679 + \cfrac{2.041063}{x + 7.885124}}}}$$



$$\begin{cases} T \\ T356(x) \cdot c(x) \\ [-5.873\ \ -218.789\ \ \text{"+"}\ \ 30\ \ \text{"blue"}] \end{cases}$$

Mutual conformity @ NBS fixed points

```
δ:= | for i ∈ 1 .. rows(XY)
    |     Δ_i := eval(round(Y_i - T356(X_i), 2))
    | augment(X, Y, Δ)
```

$$\delta = \begin{bmatrix} -5.873 & -218.789 & -0.27 \\ -5.7533 & -210.002 & -0.02 \\ -5.5356 & -195.802 & 0.04 \\ -5.3147 & -182.962 & -0.03 \\ -2.7407 & -78.476 & -0.01 \\ -1.4349 & -38.862 & 0.04 \\ 0 & 0 & -0.04 \\ 1.0679 & 26.87 & 0 \\ 4.2773 & 100 & 0.02 \\ 5.3414 & 122.37 & 0 \\ 7.0364 & 156.634 & -0.02 \\ 11.0133 & 231.9681 & 0.01 \\ 13.2188 & 271.442 & 0 \\ 16.0953 & 321.108 & 0 \\ 16.4733 & 327.502 & 0 \end{bmatrix}$$

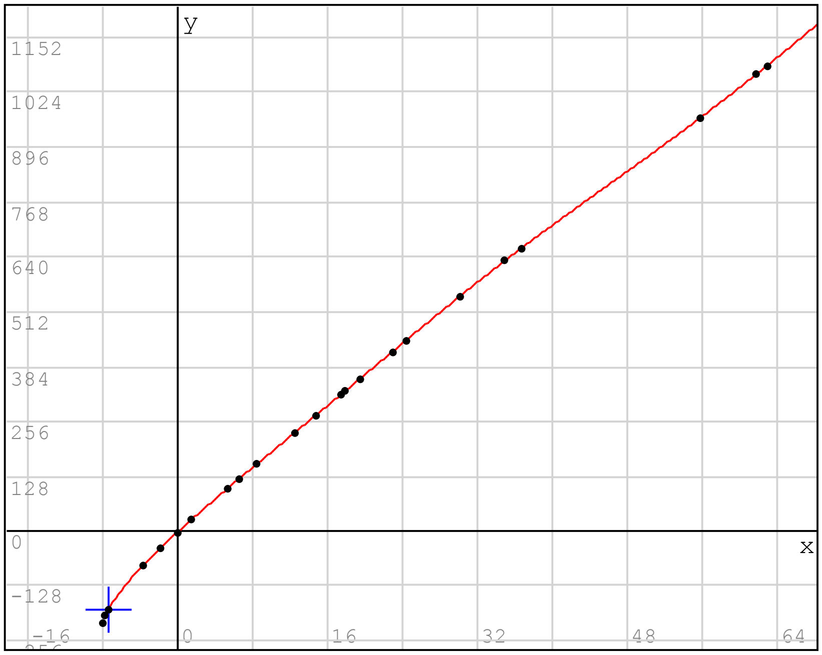
$XY := J_{2\,..\,23\;1\,..\,2}$

```
X:= col(XY, 1)
Y:= col(XY, 2)
J:= plot(J, ".", 12, "black")
```

$$c(x) := \begin{cases} 1 & \text{if } -7.4807 \le x \\ "" & \text{otherwise} \end{cases}$$

$$J1064(x) := 0.04538168 \cdot x + 12.91152 + \cfrac{98.28046}{x + 35.55858 + \cfrac{96.40391}{x + 563.9309 + \cfrac{369377.4}{x - 649.2875 - \cfrac{6.649292}{x + 12.6438}}}}$$

$J1064(x) := J1064(x) \cdot x$



```
J
J1064(x)· c(x)
[- 7.4807 - 182.962 "+" 30 "blue"]
```

Mutual conformity @ NBS fixed points

```
δ:= | for i ∈ 1 .. rows(XY)
    |    Δ i := eval(round(Y i - J1064(X i), 3))
    | augment(X, Y, Δ)
```

$$\delta = \begin{bmatrix} -7.7963 & -195.802 & -0.263 \\ -7.4807 & -182.962 & 0.036 \\ -3.7187 & -78.476 & 0 \\ -1.9063 & -38.862 & 0 \\ 0 & 0 & 0 \\ 1.3739 & 26.87 & 0 \\ 5.2677 & 100 & 0.001 \\ 6.4886 & 122.37 & 0.002 \\ 8.3743 & 156.634 & 0.002 \\ 12.5517 & 231.9681 & -0.003 \\ 14.7427 & 271.442 & -0.003 \\ 17.4928 & 321.108 & -0.001 \\ 17.8462 & 327.502 & -0.001 \\ 19.456 & 356.66 & 0.004 \\ 22.9259 & 419.58 & 0.013 \\ 24.3123 & 444.674 & 0.012 \\ 30.1095 & 548.23 & -0.022 \\ 34.9108 & 630.74 & -0.004 \\ 36.6933 & 660.37 & 0.024 \\ 55.669 & 961.93 & -0.032 \\ 61.716 & 1064.43 & 0.146 \\ 62.88 & 1084.5 & -0.116 \end{bmatrix}$$

Note: Both fixed points **[Gold, Copper]** are undetermined NBS Monograph 125 wrt to 'J' . They are just reference.
'J' fit was done Mathcad 11 [2003]. => GenfitMatrix.
This rational fraction passes very closely to the fixed points.

Part 2: Hyperfit type 'T' thermocouple

⊞— Utilities

$$T := \begin{bmatrix} -6.25629 & -268.935 \\ -6.22919 & -259.34 \\ -6.19773 & -252.87 \\ -6.17138 & -248.595 \\ -6.15358 & -246.048 \\ -5.87302 & -218.789 \\ -5.75328 & -210.002 \\ -5.53559 & -195.802 \\ -5.31472 & -182.962 \\ -2.7407 & -78.476 \\ -1.43494 & -38.862 \\ 1.0679 & 26.87 \\ 4.2773 & 100 \\ 5.3414 & 122.37 \\ 7.0364 & 156.634 \\ 11.0133 & 231.9681 \\ 13.2188 & 271.442 \\ 16.0953 & 321.108 \\ 16.4733 & 327.502 \\ 18.2179 & 356.66 \end{bmatrix}$$

$X := \mathrm{col}(T, 1)$

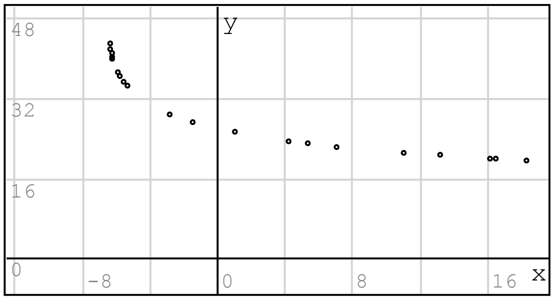
$Y := \mathrm{col}(T, 2)$

$n := \mathrm{rows}(T)$

The 'T' hyperfit is reduction technique.
The point [0,0] is removed from the fixed.
Immaterial for the suite of x*Tr(x,β)
[plot, conformity, on-line calculations]

jmG Mathcad DAEP [2003]

$$T_{reduced} := \mathrm{augment}\left(X, \overrightarrow{\frac{Y}{X}}\right)$$



$\mathrm{plot}(T_{reduced}, \text{"o"}, 4, \text{"black"})$

$$\beta := \begin{bmatrix} 25.8149913045858 \\ -12.7668104944864 \\ 1.25727409574702 \\ 3.04185012250532 \\ 0.611123673599964 \\ 0.040154623061311 \\ 0.000643845571687 3 \\ -0.46562548019054 \\ 0.030977484107051 \\ 0.119835340769071 \\ 0.027040212533505 \\ 0.002100716189884 \\ 0.000048832929912 2 \end{bmatrix}$$

$\beta := \overrightarrow{\mathrm{round}(\beta, 13)}$ roundOFF ◀

$$\mathrm{Tr}(x, \beta) := \frac{\beta_1 + \beta_2 \cdot x + \beta_3 \cdot x^2 + \beta_4 \cdot x^3 + \beta_5 \cdot x^4 + \beta_6 \cdot x^5 + \beta_7 \cdot x^6}{1 + \beta_8 \cdot x + \beta_9 \cdot x^2 + \beta_{10} \cdot x^3 + \beta_{11} \cdot x^4 + \beta_{12} \cdot x^5 + \beta_{13} \cdot x^6}$$

fit to the reduced data set.
Normalized Padé rational 6/6.
RoundOFF wisely ... rational
are sensitive to true decimals

$f(x, \beta) := x \cdot \mathrm{Tr}(x, \beta)$ reverse the fit to the original data set

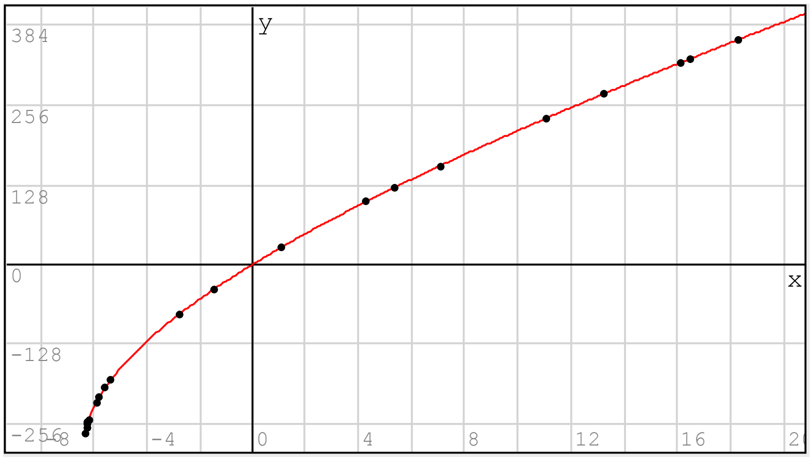
Mutual conformity @ NBS fixed points

$$c(x) := \begin{cases} 1 & \text{if } -6.25629 \le x \\ \text{""} & \text{otherwise} \end{cases}$$

$f(0, \beta) = 0$

```
δ:= | for i ∈ 1 .. rows(T)
    |     Δ_i := eval(round(Y_i − f(X_i, β), 3))
    | augment(X, Y, Δ)
```

Rational fraction to Type 'T' T/C, NBS Monograph 125



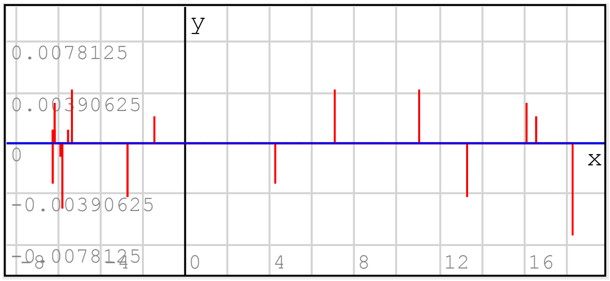
$f(X_1, \beta) = -268.936$ sanity cryogenic

$\Delta elta := \mathrm{augment}(X, \mathrm{col}(\delta, 3))$

$stem := \mathrm{Error}(\Delta elta)$

$bar := \mathrm{BarSize}(\Delta elta, 0.075)$

$datum := \mathrm{plotG}(X, 0 \cdot Y, \text{"."}, 12, \text{"black"})$



$$\delta = \begin{bmatrix} -6.2563 & -268.935 & 0.001 \\ -6.2292 & -259.34 & 0.001 \\ -6.1977 & -252.87 & -0.003 \\ -6.1714 & -248.595 & 0.001 \\ -6.1536 & -246.048 & 0.003 \\ -5.873 & -218.789 & -0.001 \\ -5.7533 & -210.002 & -0.005 \\ -5.5356 & -195.802 & 0.001 \\ -5.3147 & -182.962 & 0.004 \\ -2.7407 & -78.476 & -0.004 \\ -1.4349 & -38.862 & 0.002 \\ 1.0679 & 26.87 & 0 \\ 4.2773 & 100 & -0.003 \\ 5.3414 & 122.37 & 0 \\ 7.0364 & 156.634 & 0.004 \\ 11.0133 & 231.9681 & 0.004 \\ 13.2188 & 271.442 & -0.004 \\ 16.0953 & 321.108 & 0.003 \\ 16.4733 & 327.502 & 0.002 \\ 18.2179 & 356.66 & -0.007 \end{bmatrix}$$

β 15 D

$$\begin{bmatrix} 0 \\ 0.001 \\ -0.003 \\ 0.001 \\ 0.003 \\ -0.001 \\ -0.005 \\ 0.001 \\ 0.004 \\ -0.004 \\ 0.002 \\ 0 \\ -0.003 \\ 0 \\ 0.004 \\ 0.004 \\ -0.004 \\ 0.003 \\ 0.002 \\ -0.007 \end{bmatrix}$$

More statistical insight

SSD = "Sum Square Differences"

$$SSD := \sum_{i=1}^{n} \left(\left(Y_i - f\left(X_i , \beta \right) \right)^2 \right) = 0.000208$$

$$\begin{array}{l} \chi := \mathrm{Mean}(X) \\ \mu := \mathrm{Mean}(Y) \\ n := \mathrm{rows}(T) \end{array} \qquad \begin{bmatrix} \chi \\ \mu \\ n \end{bmatrix} = \begin{bmatrix} 1.754059 \\ -14.306345 \\ 20 \end{bmatrix}$$

Pearson’s correlation coefficient

$$corr := \frac{\dfrac{\sum_{i=1}^{n} \left(\left(X_i - \chi \right) \cdot \left(Y_i - \mu \right) \right)}{\sqrt{\sum_{i=1}^{n} \left(\left(X_i - \chi \right)^2 \right) \cdot \sum_{i=1}^{n} \left(\left(Y_i - \mu \right)^2 \right)}}}{\dfrac{\sum_{i=1}^{n} \left(\left(X_i - \chi \right) \cdot \left(\mathrm{eval}\left(f\left(X_i , \beta \right) \right) - \mu \right) \right)}{\sqrt{\sum_{i=1}^{n} \left(\left(X_i - \chi \right)^2 \right) \cdot \sum_{i=1}^{n} \left(\left(\mathrm{eval}\left(f\left(X_i , \beta \right) \right) - \mu \right)^2 \right)}}} = 0.9999999$$

correlation between [X,Y data]

correlation between [X,fitted model]

$$\begin{bmatrix} \dfrac{\sum_{i=1}^{n} \left(\left(X_i - \chi \right) \cdot \left(Y_i - \mu \right) \right)}{\sqrt{\sum_{i=1}^{n} \left(\left(X_i - \chi \right)^2 \right) \cdot \sum_{i=1}^{n} \left(\left(Y_i - \mu \right)^2 \right)}} \\ \dfrac{\sum_{i=1}^{n} \left(\left(X_i - \chi \right) \cdot \left(\mathrm{eval}\left(f\left(X_i , \beta \right) \right) - \mu \right) \right)}{\sqrt{\sum_{i=1}^{n} \left(\left(X_i - \chi \right)^2 \right) \cdot \sum_{i=1}^{n} \left(\left(\mathrm{eval}\left(f\left(X_i , \beta \right) \right) - \mu \right)^2 \right)}} \end{bmatrix} = \begin{bmatrix} 0.986337977 \\ 0.98633812 \end{bmatrix}$$

Part 3: **Ultimate 'J' fit**

⊞— 'c' normalised coefficients

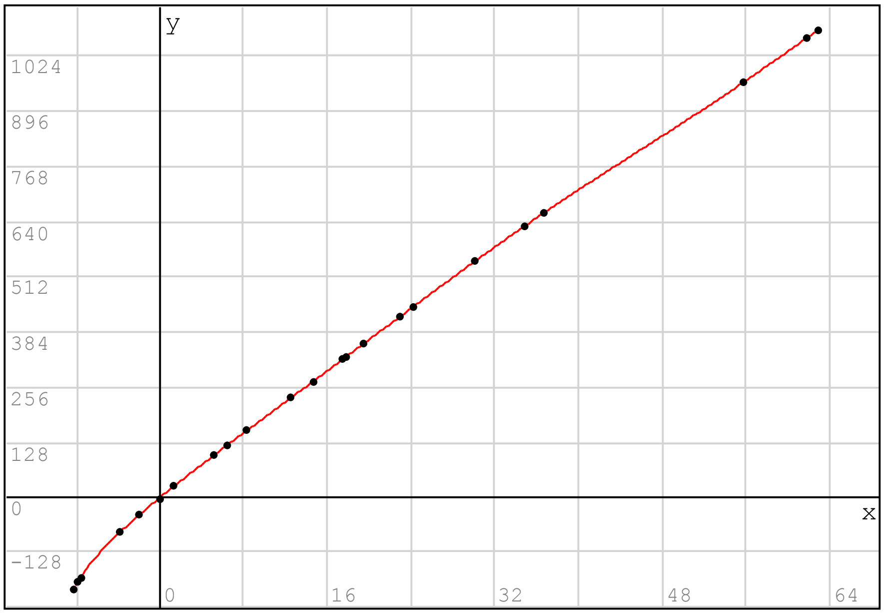
$$J1084(x) := x \cdot \frac{c_1 + c_2 \cdot x + c_3 \cdot x^2 + c_4 \cdot x^3 + c_5 \cdot x^4 + c_6 \cdot x^5 + c_7 \cdot x^6 + c_8 \cdot x^7 + c_9 \cdot x^8}{1 + c_{10} \cdot x + c_{11} \cdot x^2 + c_{12} \cdot x^3 + c_{13} \cdot x^4 + c_{14} \cdot x^5 + c_{15} \cdot x^6 + c_{16} \cdot x^7 + c_{17} \cdot x^8}$$

```
X:= col(J, 1)
Y:= col(J, 2)
J:= plot(J, ".", 12, "black")
```

Mutual conformity @ NBS fixed points

$$\Delta J1084 := \mathrm{augment}\left(X,\ Y,\ \mathrm{round}\left(\overrightarrow{\left(Y - J1084(X)\right)},\ 3\right)\right)$$

$$c(x) := \begin{cases} 1 & \text{if } \left(-8.0957 \le x\right) \wedge \left(x \le 62.88\right) \\ "" & \text{otherwise} \end{cases}$$



$$\begin{cases} J \\ J1084(x) \cdot c(x) \end{cases}$$

$$\Delta J1084 = \begin{bmatrix} -8.0957 & -210.002 & 0 \\ -7.7963 & -195.802 & 0 \\ -7.4807 & -182.962 & 0 \\ -3.7187 & -78.476 & 0 \\ -1.9063 & -38.862 & 0 \\ 0 & 0 & 0 \\ 1.3739 & 26.87 & 0 \\ 5.2677 & 100 & 0 \\ 6.4886 & 122.37 & 0 \\ 8.3743 & 156.634 & 0 \\ 12.5517 & 231.9681 & 0 \\ 14.7427 & 271.442 & 0.001 \\ 17.4928 & 321.108 & 0 \\ 17.8462 & 327.502 & 0 \\ 19.456 & 356.66 & 0 \\ 22.9259 & 419.58 & 0 \\ 24.3123 & 444.674 & 0 \\ 30.1095 & 548.23 & 0 \\ 34.9108 & 630.74 & 0 \\ 36.6933 & 660.37 & 0 \\ 55.669 & 961.93 & 0 \\ 61.716 & 1064.43 & 0 \\ 62.88 & 1084.5 & 0 \end{bmatrix}$$

Calibrate the mV/I converter

$XmV:=-100$ $XmV=\text{" °C "}$ $mV:=1$

$\text{solve}\left(J1084\left(mV\right)-XmV,\ mV,\ -5,\ -4\right)=-4.6322\ mV$

$J1084\left(-4.6309\right)=-99.97$

Low range: -100°C => -4.6322 mV

$XmV:=200$ $XmV=\text{" °C "}$ $mV:=1$

$\text{solve}\left(J1084\left(mV\right)-XmV,\ mV,\ 0,\ 20\right)=10.7765\ mV$

$J1084\left(10.7768\right)=200$

High range: 200°C => 10.7765 mV

Part 4: => **'J'cryogenic [-210.002...26.87]**
'T'cryogenic [-268.935...26.87

$$\mathrm{Jcryo}(x) := 2319.64 - \frac{286005.3}{x + 123.8632 - \frac{5.737392}{x + 10.21686}}$$

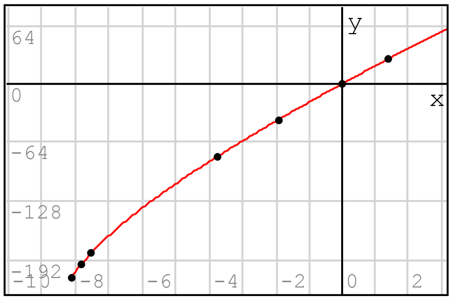
$$J := J_{1..7\ 1..2}$$

$$J = \begin{bmatrix} -8.0957 & -210.002 \\ -7.7963 & -195.802 \\ -7.4807 & -182.962 \\ -3.7187 & -78.476 \\ -1.9063 & -38.862 \\ 0 & 0 \\ 1.3739 & 26.87 \end{bmatrix}$$

$$\left| \begin{array}{l} X := \mathrm{col}(J, 1) \\ Y := \mathrm{col}(J, 2) \end{array} \right.$$

$$\mathrm{Jcryo} := \mathrm{plot}(J, ".", 12, "black")$$

$$c(x) := \begin{cases} 1 & \text{if } -8.0957 \le x \\ "" & \text{otherwise} \end{cases}$$



$$\begin{cases} \mathrm{Jcryo} \\ \mathrm{Jcryo}(x) \cdot c(x) \end{cases}$$

Mutual conformity @ NBS fixed points

$$\Delta\mathrm{Jcryo} := \mathrm{augment}\left(X, Y, \overrightarrow{\mathrm{round}\left(\overrightarrow{(Y - \mathrm{Jcryo}(X))}, 2\right)}\right)$$

$$\Delta\mathrm{Jcryo} = \begin{bmatrix} -8.0957 & -210.002 & -0.02 \\ -7.7963 & -195.802 & 0.07 \\ -7.4807 & -182.962 & -0.05 \\ -3.7187 & -78.476 & 0.02 \\ -1.9063 & -38.862 & -0.02 \\ 0 & 0 & -0.08 \\ 1.3739 & 26.87 & 0 \end{bmatrix}$$

==

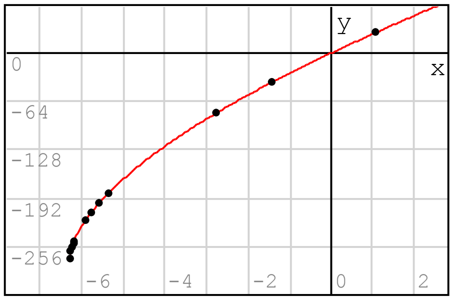
$$T := T_{1..12\ 1..2}$$

$$\mathrm{Tcryo}(x) := 18.41415 \cdot x + 76.66062 - \frac{791.578}{x + 10.35533 - \frac{0.1977402}{x + 6.656865 - \frac{0.005441432}{x + 6.30942}}}$$

$$\left| \begin{array}{l} X := \mathrm{col}(T, 1) \\ Y := \mathrm{col}(T, 2) \end{array} \right.$$

$$\mathrm{Tcryo} := \mathrm{plot}(T, ".", 12, "black")$$

$$c(x) := \begin{cases} 1 & \text{if } -6.2563 \le x \\ "" & \text{otherwise} \end{cases}$$



$$\begin{cases} \mathrm{Tcryo} \\ \mathrm{Tcryo}(x) \cdot c(x) \end{cases}$$

Mutual conformity @ NBS fixed points

$$\Delta\mathrm{Tcryo} := \mathrm{augment}\left(X, Y, \overrightarrow{\mathrm{round}\left(\overrightarrow{(Y - \mathrm{Tcryo}(X))}, 2\right)}\right)$$

$$\Delta\mathrm{Tcryo} = \begin{bmatrix} -6.2563 & -268.935 & 0 \\ -6.2292 & -259.34 & 0.02 \\ -6.1977 & -252.87 & -0.06 \\ -6.1714 & -248.595 & 0 \\ -6.1536 & -246.048 & 0.04 \\ -5.873 & -218.789 & 0.01 \\ -5.7533 & -210.002 & -0.03 \\ -5.5356 & -195.802 & -0.01 \\ -5.3147 & -182.962 & 0.03 \\ -2.7407 & -78.476 & -0.02 \\ -1.4349 & -38.862 & 0.02 \\ 1.0679 & 26.87 & 0 \end{bmatrix}$$

About this document . . .

1. You will read all sorts of empirical jargon from literature [outdated, repeated, not CAS]. This document is a project guide . Many more types T/C exist, not explored.
All in all, **T, J** cover most industrial processes. If you need down to absolute '0', you may consider cryogenic sensors in that particular short low range, eventually RTD Pt100.

2. This document exemplifies fitting the NBS fixed points. For an accurate measurement from T/C suppliers, you would need to simulate fixed points from thermal bath and run the **Mathcad GenfitMatrix** of which the core algorithm is the built-in **genfit(X,Y,guesses,F)** where X,Y are the data set, and F the model function. Unfortunately and up todate, such an algorithm is not available yet in Smath. If not from public, will have to be user coded.

Part 5: => Prefered 'J' [-210.002 ... 500 °C]

⊞— Genfit Utilities

$$XY:=\begin{bmatrix} -8.0957 & -210.002 \\ -7.7963 & -195.802 \\ -7.4807 & -182.962 \\ -3.7187 & -78.476 \\ -1.9063 & -38.862 \\ 0 & 0 \\ 1.3739 & 26.87 \\ 5.2677 & 100 \\ 6.4886 & 122.37 \\ 8.3743 & 156.634 \\ 12.5517 & 231.9681 \\ 14.7427 & 271.442 \\ 17.4928 & 321.108 \\ 17.8462 & 327.502 \\ 19.456 & 356.66 \\ 22.9259 & 419.58 \\ 24.3123 & 444.674 \end{bmatrix}$$

$$f(x,\beta):=\frac{\beta_1+\beta_2\cdot x+\beta_3\cdot x^2+\beta_4\cdot x^3+\beta_5\cdot x^4}{1+\beta_6\cdot x+\beta_7\cdot x^2+\beta_8\cdot x^3}$$

Padé rational fraction
..... NORMALIZED .....
leading 1 in denom

$t0:=\mathrm{time}(1)$

$\varphi(x,\beta):=\varphi(x,\beta,f,8)$ conjugate gradient

$X:=\mathrm{col}(XY,1)$
$Y:=\mathrm{col}(XY,2)$
$n:=\mathrm{rows}(XY)$

appVersion(3) = "0.98.6179"
Timing = "17 sec"

```
β:= | β_1:= [-0.5; 20; -1.5; 0.05; 0.01; 0; 0; 0]
    | iter:= 13
    | i:= 1
    | while i ≤ iter
    |     | β_(i+1):= Minimize(XY, f, φ, β_i)
    |     | i:= i + 1
    | β:= β_iter
```

$$\begin{bmatrix} -0.5 \\ 20 \\ -1.5 \\ 0.05 \\ 0.01 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\beta:=\beta$

$$\beta=\begin{bmatrix} 0.022892692 \\ 19.863847509 \\ 2.921904294 \\ 0.154170228 \\ 0.005263643 \\ 0.159363729 \\ 0.00876669 \\ 0.000280425 \end{bmatrix}$$

$\mathrm{time}(1)-t0=19\ s$

SSD = "Sum Square Differences"

$$SSD:=\sum_{i=1}^{n}\left(\left(Y_i-f\left(X_i,\beta\right)\right)^2\right)=0.02$$

Normalized rational Padé converted to continued J_Fraction [Maple].

$$J(x) := 18.770251 \cdot x - 37.025038 + \cfrac{910.027136}{x + 3.85542 + \cfrac{317.453992}{x + 17.936706 - \cfrac{24.686096}{x + 9.470053}}}$$

```
FIT:= | U:= - 8.1 , - 8 .. 30
      | for  i ∈ 1 .. rows(U)
      |     vy_i := eval(J(U_i))
      | augment(U , vy)
```

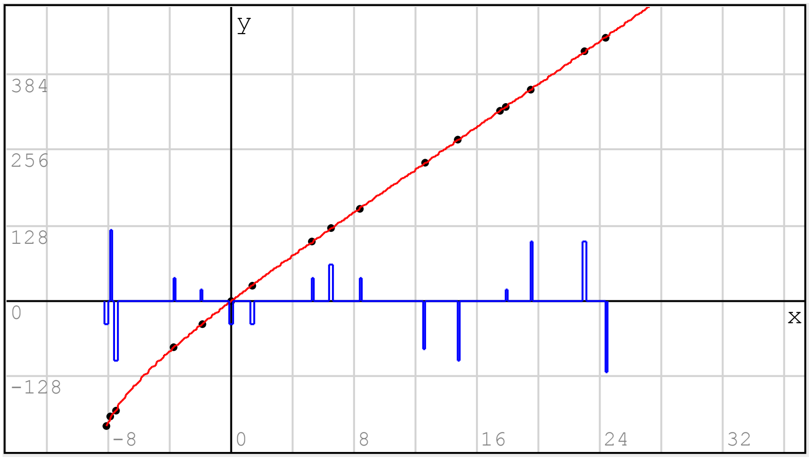
Mutual conformity @ NBS fixed points

```
δ:= | for  i ∈ 1 .. rows(XY)
    |     Δ_i := eval(round(Y_i - J(X_i) , 2))
    | augment(X , Y , Δ)
```

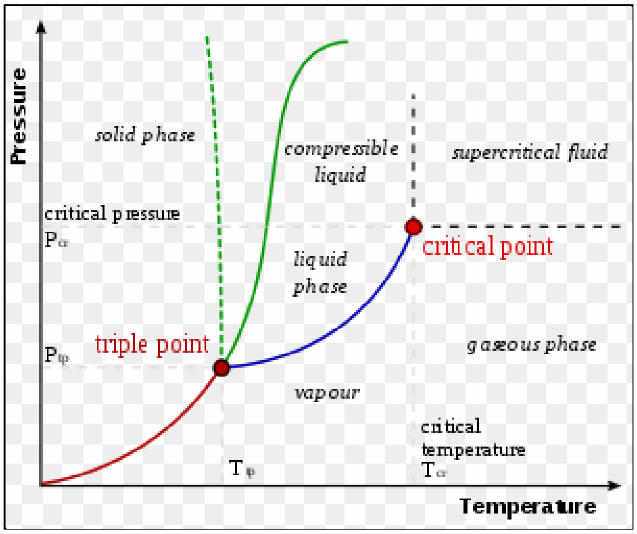
```
Δelta:= augment(X , col(δ , 3)·2000)
bar:= BarSize(Δelta , 0.075)
data:= plot(XY , "." , 12 , "black")
```

$$\delta = \begin{bmatrix} -8.0957 & -210.002 & -0.02 \\ -7.7963 & -195.802 & 0.06 \\ -7.4807 & -182.962 & -0.05 \\ -3.7187 & -78.476 & 0.02 \\ -1.9063 & -38.862 & 0.01 \\ 0 & 0 & -0.02 \\ 1.3739 & 26.87 & -0.02 \\ 5.2677 & 100 & 0.02 \\ 6.4886 & 122.37 & 0.03 \\ 8.3743 & 156.634 & 0.02 \\ 12.5517 & 231.9681 & -0.04 \\ 14.7427 & 271.442 & -0.05 \\ 17.4928 & 321.108 & 0 \\ 17.8462 & 327.502 & 0.01 \\ 19.456 & 356.66 & 0.05 \\ 22.9259 & 419.58 & 0.05 \\ 24.3123 & 444.674 & -0.06 \end{bmatrix}$$

Type 'J' Thermocouple fit to NBS Monograph 125



⊞ Convert Continued fraction



The procedure => 'J' [-210.002 ... 500 °C]

... suggests that if you have bath tested the fixed points, it might be possible to evaluate the **β**'s ... thus, **personalized** the concerned 'J' T/C for high accuracy.

At the triple point [0.01 °C], all three phases can coexist.
Water critical point : However, the liquid-vapor boundary terminates in an endpoint at some critical temperature Tc and critical pressure pc. This is the critical point.
In water, the critical point occurs at around
647 K (374 °C) and 22.064 MPa (3200 psia or 218 atm)

Sub-Cryogenic sensor CY7

⊞— plot(data,char,size,clr)... CY7

Suggested renovation CY7 ================================

$$cry\_1(x) := -20.36985 \cdot x + 31.19133 + \frac{3.890508}{x - 0.935192 + \frac{0.001116245}{x - 1.099057}}$$

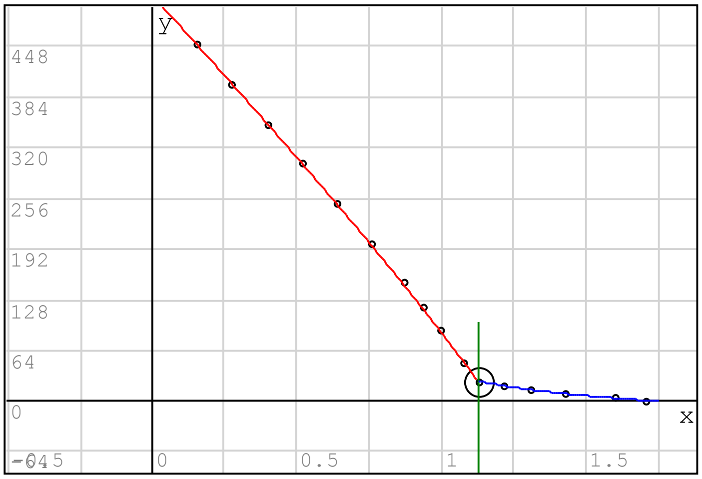
break= 1.1246

support:= plot(CY7, "o", 5, "bla

$$cry\_2(x) := -380 \cdot x + 546.8 + \frac{57}{x - 1.5 + \frac{0.2}{x - 2 - \frac{0.04}{x - 1 + \frac{5}{x - 1}}}}$$

$$c1(x) := \begin{cases} 1 & \text{if } (1.1246 \le x) \wedge (x \le 1.75) \\ "" & \text{otherwise} \end{cases}$$

$$c2(x) := \begin{cases} 1 & \text{if } (0 \le x) \wedge (x \le 1.1246) \\ "" & \text{otherwise} \end{cases}$$



$$\begin{bmatrix} cry\_1(1.1246) \\ cry\_2(1.1246) \end{bmatrix} = \begin{bmatrix} 25.0 \\ 25.0 \end{bmatrix}$$

Comment . . .

The phenomenon behaves like discontinuous.
As published, high order Chebychev Omega aren't much inspiring ... [handling, computation load].
Rational fraction is a good candidate for interpolating each segment. In this proposal, we consider the break point as an **Act of God** . More information might help to conclude/renovate this type of low cryogenic sensor.

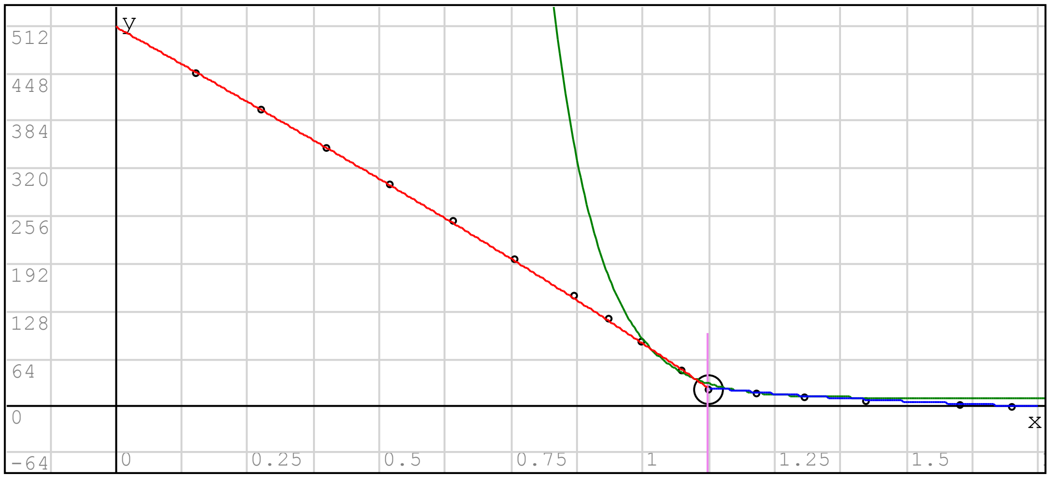
$$XY := \begin{bmatrix} 1.3042 & 15 \\ 1.2144 & 20 \\ 1.1246 & 25 \\ 1.0705 & 50 \\ 0.9956 & 90 \end{bmatrix} \qquad \beta := \begin{bmatrix} 11.94 \\ 13.831 \\ 1.155 \\ 0.089 \end{bmatrix}$$

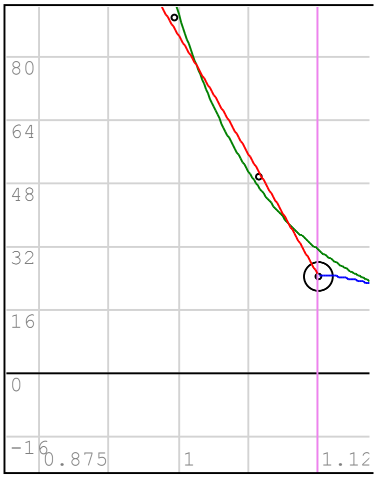
$$f(x, \beta) := \beta_1 + \beta_2 \cdot \exp\left(-\left(\frac{x - \beta_3}{\beta_4}\right)\right)$$

model to smooth the break region

XY:= plot(XY, "o", 5, "black")

$$m(x) := \begin{cases} 1 & \text{if } (0.975 \le x) \wedge (x \le 1.5) \\ "" & \text{otherwise} \end{cases}$$





$$jct := \text{solve}(cry\_2(x) - f(x, \beta) = 0, x, 1.0625, 1.125) = 1.097816$$

$$jct := \text{solve}(cry\_1(x) - f(x, \beta) = 0, x, 1.125, 1.25) = 1.183984$$

$$\text{solve}(cry\_1(x) = 0, x, 1.7, 1.85) = 1.7618$$

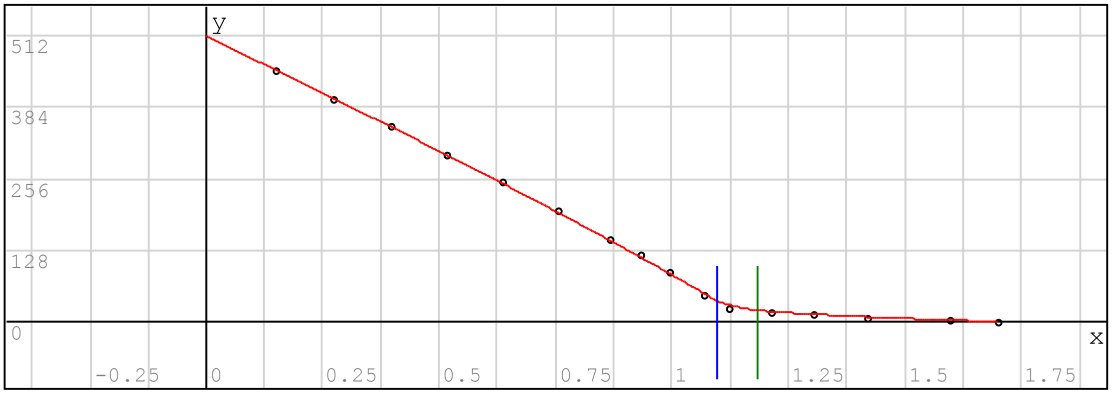
possible CY7 0 °K extrapolation

$$CY7(x) := \begin{cases} cry\_2(x) & \text{if } (0 \le x) \wedge (x \le 1.097816) \\ f(x, \beta) & \text{if } (1.097816 \le x) \wedge (x \le 1.183984) \\ cry\_1(x) & \text{otherwise} \end{cases}$$

$$jct := \begin{bmatrix} 1.097816 \\ 1.183984 \end{bmatrix}$$

$$m(x) := \begin{cases} 1 & \text{if } (0 \le x) \wedge (x \le 1.7) \\ "\," & \text{otherwise} \end{cases}$$

$$CY7(1.7618) = 0$$



$$CY7 = \begin{bmatrix} 1.6981 & 1.4 \\ 1.5978 & 5 \\ 1.4201 & 10 \\ 1.3042 & 15 \\ 1.2144 & 20 \\ 1.1246 & 25 \\ 1.0705 & 50 \\ 0.9956 & 90 \\ 0.9338 & 120 \\ 0.8687 & 150 \\ 0.7555 & 200 \\ 0.6384 & 250 \\ 0.5189 & 300 \\ 0.3978 & 350 \\ 0.2746 & 400 \\ 0.1498 & 450 \end{bmatrix}$$

Read more . . .

I'm not aware of industrial process in the sub-cryogenic of Pt 100 [13.8 °K, -259.3467 °C] . To meet the demand starting ~ 1 °K, @ best accuracy on longer range, a dual installation is recommanded ... CY7 switching around 20 °K to Pt 100. No more detail about installation. This technical suggestion is subject to project requirements.

$$\begin{cases} V := \text{col}(CY7, 1) \\ K := \text{col}(CY7, 2) \end{cases}$$

Forward voltage [V]
Temperature [°K]

```
conformity:= for i ∈ 1 .. rows(V)
                 cy7_i := eval(CY7(V_i))
             augment(V, K, K - round(cy7, 1)→)
```

$$conformity = \begin{bmatrix} 1.6981 & 1.4 & -0.3 \\ 1.5978 & 5 & 0.5 \\ 1.4201 & 10 & -0.2 \\ 1.3042 & 15 & 0 \\ 1.2144 & 20 & 0.1 \\ 1.1246 & 25 & -6.4 \\ 1.0705 & 50 & -1.6 \\ 0.9956 & 90 & 2.5 \\ 0.9338 & 120 & 3.7 \\ 0.8687 & 150 & 3.8 \\ 0.7555 & 200 & 3.2 \\ 0.6384 & 250 & 2.3 \\ 0.5189 & 300 & 1.4 \\ 0.3978 & 350 & 0.8 \\ 0.2746 & 400 & 0 \\ 0.1498 & 450 & -0.8 \end{bmatrix}$$

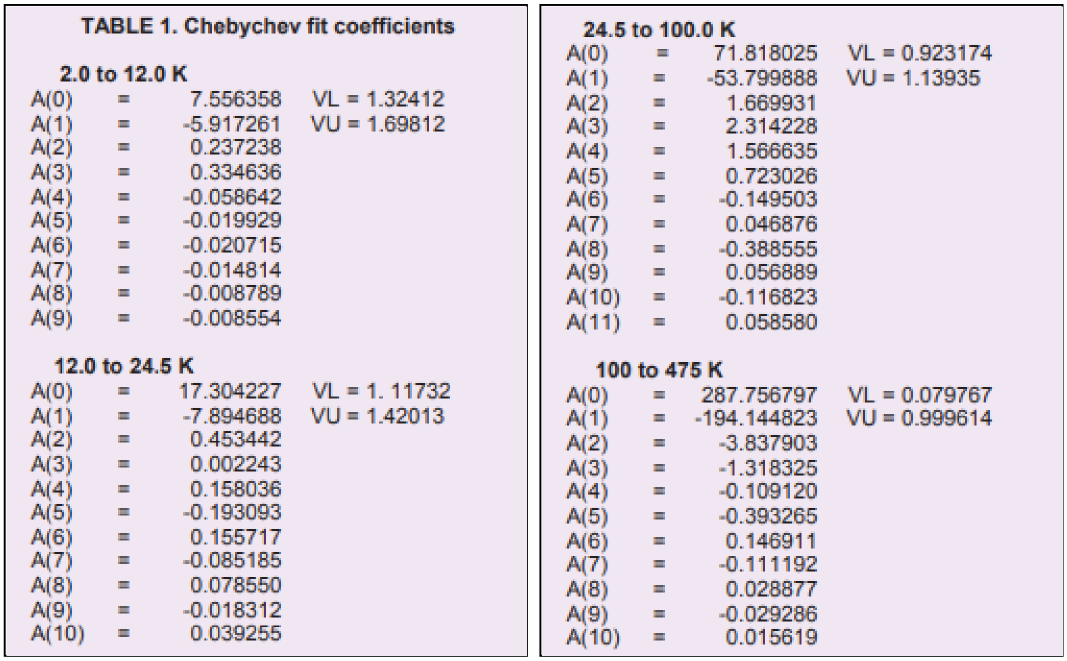
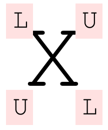
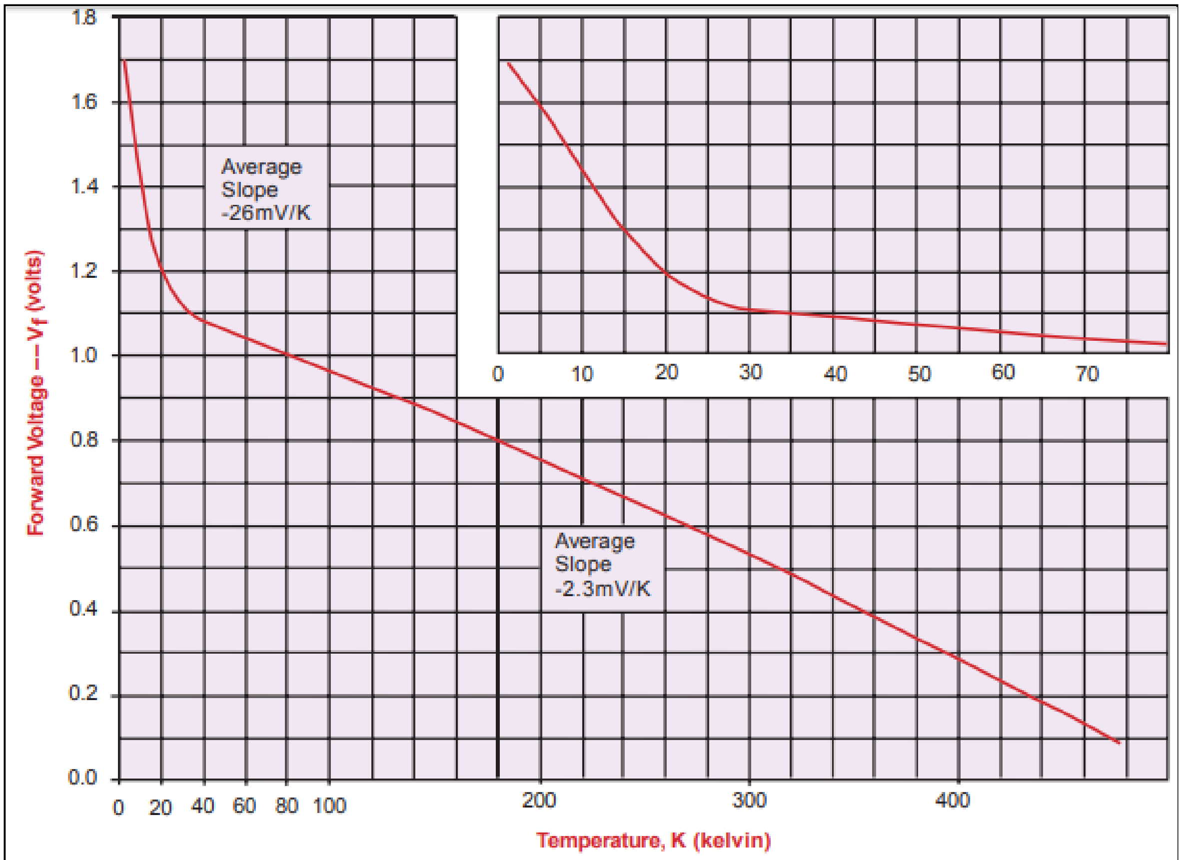


TABLE 1. Chebychev fit coefficients

2.0 to 12.0 K			
A(0)	=	7.556358	VL = 1.32412
A(1)	=	-5.917261	VU = 1.69812
A(2)	=	0.237238	
A(3)	=	0.334636	
A(4)	=	-0.058642	
A(5)	=	-0.019929	
A(6)	=	-0.020715	
A(7)	=	-0.014814	
A(8)	=	-0.008789	
A(9)	=	-0.008554	
12.0 to 24.5 K			
A(0)	=	17.304227	VL = 1. 11732
A(1)	=	-7.894688	VU = 1.42013
A(2)	=	0.453442	
A(3)	=	0.002243	
A(4)	=	0.158036	
A(5)	=	-0.193093	
A(6)	=	0.155717	
A(7)	=	-0.085185	
A(8)	=	0.078550	
A(9)	=	-0.018312	
A(10)	=	0.039255	
24.5 to 100.0 K			
A(0)	=	71.818025	VL = 0.923174
A(1)	=	-53.799888	VU = 1.13935
A(2)	=	1.669931	
A(3)	=	2.314228	
A(4)	=	1.566635	
A(5)	=	0.723026	
A(6)	=	-0.149503	
A(7)	=	0.046876	
A(8)	=	-0.388555	
A(9)	=	0.056889	
A(10)	=	-0.116823	
A(11)	=	0.058580	
100 to 475 K			
A(0)	=	287.756797	VL = 0.079767
A(1)	=	-194.144823	VU = 0.999614
A(2)	=	-3.837903	
A(3)	=	-1.318325	
A(4)	=	-0.109120	
A(5)	=	-0.393265	
A(6)	=	0.146911	
A(7)	=	-0.111192	
A(8)	=	0.028877	
A(9)	=	-0.029286	
A(10)	=	0.015619	



$$\frac{(x-L)-(U-x)}{(U-L)}$$



Standard Temperature Response (Curve 10) for CY7 Series Sensors.
All Sensors Track this Curve Within Specified Tolerance Bands.

$CY7 := CY7_{1..7\ 1..2}$

$\begin{cases} V := col(CY7, 1) \\ K := col(CY7, 2) \end{cases}$

Forward [V] ... [°K]...

$$CY7 = \begin{bmatrix} 1.6981 & 1.4 \\ 1.5978 & 5 \\ 1.4201 & 10 \\ 1.3042 & 15 \\ 1.2144 & 20 \\ 1.1246 & 25 \\ 1.0705 & 50 \end{bmatrix}$$

Possible reconciliation CY7

That two [2] ChebyShev have to cover such a small range [2K .. 24.5 K] is a strict indication of poorly collected data or CY7 not reflecting the physics of this subcryogenic range. To conclude CY7, up to better experimental CY7 data => switch CY7 to Pt100 or type 'T' @ either 1.2144 or 1.2146 and ignore the upper CY7 [°K]

This subcryo CY7 function flies in the blue @ 'x' ≤ 1.1246

$$sub\_CY7(x) := -20.36985 \cdot x + 31.19133 + \cfrac{3.890508}{x - 0.935192 + \cfrac{0.001116245}{x - 1.099057}}$$

$ITS := solve(sub\_CY7(x) - 13.8033 = 0, x, 1.3, 1.4) = 1.330705$

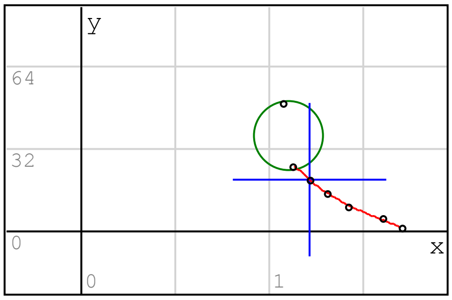
determine the switch point

$$\begin{cases} sub := plot(CY7, \text{"o"}, 5, \text{"black"}) \\ switch := [1.2144\ \ sub\_CY7(1.2144)\ \ \text{"+"}\ \ 100\ \ \text{"blue"}] \\ exclusive := [1.09755\ \ 37.5\ \ \text{"o"}\ \ 60\ \ \text{"green"}] \end{cases}$$

sub-range to sub_CY7(x)
proposed switch => Pt100
exclusive region of sub_CY7(x)

$$c(x) := \begin{cases} 1 & \text{if } (1.1246 \le x) \wedge (x \le 1.6981) \\ \text{""} & \text{otherwise} \end{cases}$$

sub-cryogenic CY7 ... switch
@ 1.2144 = 20 °K [-253.15 °C]



$$\overrightarrow{K - sub\_CY7(V)} = \begin{bmatrix} -0.29 \\ 0.5 \\ -0.23 \\ -0.01 \\ 0.08 \\ 0.03 \\ 0.18 \end{bmatrix}$$

$$\begin{bmatrix} sub\_CY7(ITS) \\ sub\_CY7(1.1246) \end{bmatrix} = \begin{bmatrix} 13.8 \\ 24.97 \end{bmatrix}$$

°Kelvin

$(-268.935)\ °C = 4.215\ K$

type 'T'

$(-248.5939)\ °C = 24.5561\ K$

$$\begin{bmatrix} 13.8033\ K \\ 20\ K \end{bmatrix} = \begin{bmatrix} -259.3467 \\ -253.15 \end{bmatrix} °C$$

$$ITS\_1990(°C) = \left[\begin{bmatrix} -259.3467 \\ -248.5939 \\ -218.7916 \\ -189.3442 \\ -38.8344 \\ 0.01 \\ 29.7646 \\ 156.5985 \\ 231.928 \\ 419.527 \\ 660.323 \\ 961.78 \end{bmatrix} \begin{bmatrix} \text{"TP Hydrogen"} \\ \text{"TP Neon"} \\ \text{"TP Oxygen"} \\ \text{"TP Argon"} \\ \text{"TP Mercury"} \\ \text{"TP Water"} \\ \text{"MP Gallium"} \\ \text{"FP Indium"} \\ \text{"FP Tin"} \\ \text{"FP Zinc"} \\ \text{"FP Aluminum"} \\ \text{"FP Silver"} \end{bmatrix} \right]$$

ITS_1990 Bible points

The triple points of several substances are used to define points in the ITS-90 international temperature scale, ranging from the **triple point of hydrogen (13.8033 K) . . . -259.3467 °C** to the triple point of water (273.16 K, 0.01 °C, or 32.018 °F)