$appVersion(4) = "1.0.8348.30405"$

$appVersion(-4) = "1.0.8348.30405"$

Program 1 : Pairing non-diagonal indices of stiffness matrix

$$Pair\_STF(B\#) := \left| \begin{array}{l} nr\# := rows(B\#) - 1 \\ \text{for } k \in [1..(nr\#)] \\ \quad \left| \; Z\#_k := stack\left(B\#_k, B\#_{k+1}\right) \right. \\ Z\# \end{array} \right.$$

Define

$$S_{BEAM} := \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

Program 2 : To find the Stiffness Natrix : Calls Program 1
fdn#=1 if foundatio column base is FIXED ", fdn#=0.75 if foundation column base is PINNED

$$Find\_K\left(L\#, I\#_{UC}, I\#_{LC}, I\#_{beam}, H\#_{UC}, H\#_{LC}, fdn\#\right) := \left| \begin{array}{l} nr := rows(L\#) + 1 \\ \text{for } j \in [1..(nr)] \\ \quad \left| \; M\#_j := 4 \cdot E \cdot \begin{cases} \dfrac{I\#_{UC_j}}{H\#_{UC_j}} + \dfrac{I\#_{LC_j} \cdot fdn\#}{H\#_{LC_j}} + \dfrac{I\#_{beam_j}}{L\#_j} & \text{if } j = 1 \\ \dfrac{I\#_{UC_j}}{H\#_{UC_j}} + \dfrac{I\#_{LC_j} \cdot fdn\#}{H\#_{LC_j}} + \dfrac{I\#_{beam_j}}{L\#_{j-1}} + \dfrac{I\#_{beam_j}}{L\#_j} & \text{if } 1 < j < nr \\ \dfrac{I\#_{UC_j}}{H\#_{UC_j}} + \dfrac{I\#_{LC_j} \cdot fdn\#}{H\#_{LC_j}} + \dfrac{I\#_{beam_{j-1}}}{L\#_{j-1}} & \text{otherwise} \end{cases} \right. \\ (STF := Diag(M\#)) = \text{"Create diagonal matrix"} \\ C\# := [1..nr] \\ (D\# := Pair\_STF(C\#)) = \text{"Indices of non-diagonal elements"} \\ Clear(j) \\ \text{for } j \in [1..rows(L\#)] \\ \quad \left| \begin{array}{l} \left(M2\#_j := 2 \cdot E \cdot \dfrac{I\#_{beam_j}}{L\#_j}\right) = \text{"Non-diagonal elements"} \\ \left(r_j := D\#_{j_1}\right) = \text{"Row index of non-diagonal elements"} \\ \left(c_j := D\#_{j_2}\right) = \text{"Column index of non-diagonal elements"} \\ \left(STF_{r_j c_j} := M2\#_j\right) = \text{"Assign appropriate M2\# to STF"} \\ \left(STF_{c_j r_j} := M2\#_j\right) = \text{"Assign appropriate M2\# to STF"} \end{array} \right. \\ STF \end{array} \right.$$

PROGRAM 3 : To find joint moments & ST

$$Find\_ST\#(Z, aa(\blacksquare), kk) := \left| \begin{array}{l} j := [1..rows(Z)] \\ jm_j := aa\left(Z_j\right) \\ st_j := -kk^{-1} \cdot jm_j \\ st \end{array} \right.$$

Program 4 : Any function can be passed to this program

$$Arrange(Z\#, a(\blacksquare)) := \left| \begin{array}{l} j := [1..rows(Z\#)] \\ D_j := a\left(Z\#_j\right) \end{array} \right.$$

Program 5 : Find lower column moments. Valid even if foundation columns have equal or diff heights

$$Find\_MLC(st, d) := \left| \begin{array}{l} \text{for } j \in [1..rows(st)] \\ \quad \left| \begin{array}{l} a\# := st_j \\ A\#_j := \overrightarrow{a\# \cdot d} \end{array} \right. \\ A\# \end{array} \right.$$

Program 6 : Calls Program 4

$$Pair(B\#) := \left| \begin{array}{l} nrr := \overrightarrow{rows(B\#)}_1 - 1 \\ \text{for } k \in [1..(nrr)] \\ \quad \left| \; Z\#_k := \begin{bmatrix} YY_k \\ YY_{k+1} \end{bmatrix} \right. \\ FF(YY) := Z\# \\ Arrange(B\#, FF(YY)) \end{array} \right.$$

Program 7 : Main program to find support moments, upper & lower column moments
Calls Programs 3, 4, 5 & 6

```
NEW_BMM(ff#, K#, h#_UC, h#_LC, I_b, I_C, I_L, fdn#, L#) :=
  for j ∈ [1..cols(ff#)]
    fem#_j := (1/12)·(ff#_<1 j> · L#^2)→
  nrr := rows(fem#)→_1 + 1
  for j ∈ [1..nrr]
    M#_j := | −Y_j            if  j = 1
            | Y_(j−1) − Y_j   if  1 < j < nrr
            | Y_(nrr−1)       otherwise
  F(Y) := M#
  st := Find_ST#(fem#, F(Y), K#)
  stt := Pair(st)
  for j ∈ [1..cols(ff#)]
    for k ∈ [1..rows(L)]
      M_sup_(j k) := ( stt_(j k) · E · I_b_k / L_k )^T · S_BEAM + [ −fem#_(j k) ; fem#_(j k) ]^T
  Col_bott# := (I_L / h#_LC)→ · 4 · E · fdn#
  M_LCC := Find_MLC(st, Col_bott#)
  Col_upp# := (I_C / h#_UC)→ · 4 · E
  M_UCC := Find_MLC(st, Col_upp#)
  augment(M_sup, M_UCC, M_LCC)
```

Define

$kNm := kN\,m$

Assume

$E := 1\ MPa$

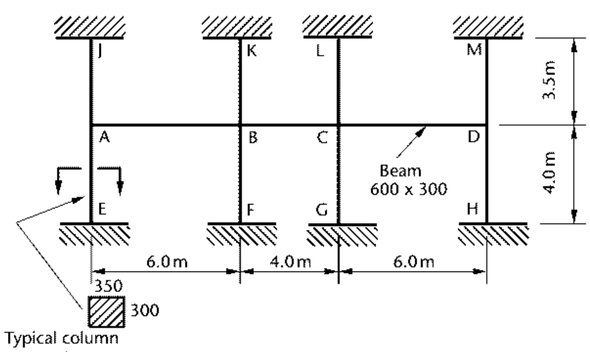
Assume Young's Modulus for RC: This is not really required, but used only for consistent units in intermediate calculations. We can use any numerical value. **Ex E = 1 MPa**

Ex 3.3 - Page 44 : Reinforced concrete design to Eurocode 2 : *Mosely, Bungey & Hulse - 7th Ed - pal grave macmillan*

EXAMPLE 3.3

Analysis of a substitute frame

The substitute frame shown in figure 3.12 is part of the complete frame in figure 3.10. The characteristic actions carried by the beams are permanent actions (including self-weight) $G_k = 25$ kN/m, and variable action, $Q_k = 10$ kN/m, uniformly distributed along the beam. The analysis of the subframe will be carried out by moment distribution: thus the member stiffnesses and their relevant distribution factors are first required.



Member Stiffness (ST)

cond = 0.75 when Pinned at base.

cond = 1 when fixed at both ends.

This program can handle both cases as shown below.

$$cond := \begin{bmatrix} \text{"Pinned"} & 0.75 \\ \text{"Fixed"} & 1 \end{bmatrix}$$

○ Pinned ◉ Fixed

FDN

DATA

4 upper columns, 4 lower columns and 3 beams

Foundation Columns Fixed at Base

$FDN = 1$

Column

$$b_{UC} := \begin{bmatrix} 300 \\ 300 \\ 300 \\ 300 \end{bmatrix} \text{mm}$$

$$h_{UC} := \begin{bmatrix} 350 \\ 350 \\ 350 \\ 350 \end{bmatrix} \text{mm}$$

$$b_{LC} := \begin{bmatrix} 300 \\ 300 \\ 300 \\ 300 \end{bmatrix} \text{mm}$$

$$h_{LC} := \begin{bmatrix} 350 \\ 350 \\ 350 \\ 350 \end{bmatrix} \text{mm}$$

$$H_{UC} := \begin{bmatrix} 3.5 \\ 3.5 \\ 3.5 \\ 3.5 \end{bmatrix} \text{m}$$

$$H_{LC} := \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix} \text{m}$$

Beam

$$b_{beam} := \begin{bmatrix} 300 \\ 300 \\ 300 \end{bmatrix} \text{mm}$$

$$h_{beam} := \begin{bmatrix} 600 \\ 600 \\ 600 \end{bmatrix} \text{mm}$$

$$L := \begin{bmatrix} 6 \\ 4 \\ 6 \end{bmatrix} \text{m}$$

Define

$kNm := \text{kN m}$

Assume

$E := 1 \text{ MPa}$

Flanged Beams can be effective only in case of sagging as the concrete is able to take compression. In limit state design we neglect the tensile strength of concrete. As a result, at supports, continuous **'T' beams should be designed as a rectangular beam** ***(with width equal to the web width)***.

Defie

$$Line_{zero} := \begin{bmatrix} 0 & 0 \\ \sum\left(\frac{L}{\text{m}}\right) & 0 \end{bmatrix}$$

Define

$$LL_{[1..rows(L)]} := 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Moment of Inertia

Upper Columna

$$I_{UC} := \overrightarrow{\frac{1}{12} \cdot b_{UC} \cdot \left(h_{UC}\right)^3} = \begin{bmatrix} 1.0719 \cdot 10^9 \\ 1.0719 \cdot 10^9 \\ 1.0719 \cdot 10^9 \\ 1.0719 \cdot 10^9 \end{bmatrix} \text{mm}^4$$

Lower Columna

$$I_{LC} := \overrightarrow{\frac{1}{12} \cdot b_{LC} \cdot \left(h_{LC}\right)^3} = \begin{bmatrix} 1.0719 \cdot 10^9 \\ 1.0719 \cdot 10^9 \\ 1.0719 \cdot 10^9 \\ 1.0719 \cdot 10^9 \end{bmatrix} \text{mm}^4$$

Beams

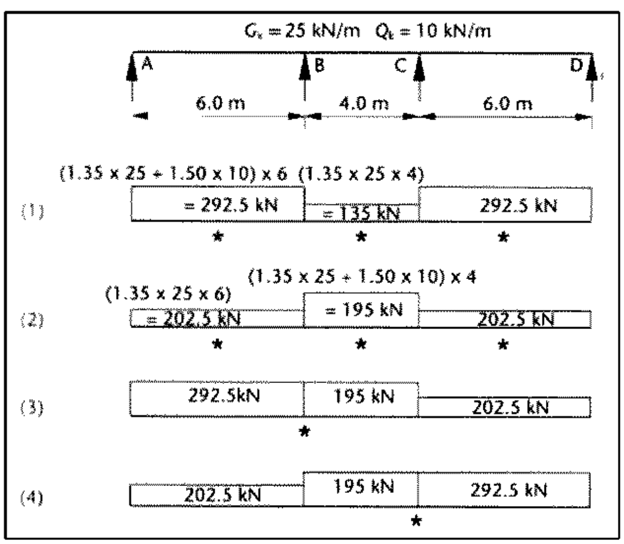
$$I_{beam} := \overrightarrow{\frac{1}{12} \cdot b_{beam} \cdot \left(h_{beam}\right)^3} = \begin{bmatrix} 5.4 \cdot 10^9 \\ 5.4 \cdot 10^9 \\ 5.4 \cdot 10^9 \end{bmatrix} \text{mm}^4$$

$FDN = 1$

Call Program 2 to find the Stiffness Matrix of the Sustitute Frame

$$K1 := Find\_K\left(L, I_{UC}, I_{LC}, I_{beam}, H_{UC}, H_{LC}, FDN\right) = \begin{bmatrix} 5.8969 & 1.8 & 0 & 0 \\ 1.8 & 11.2969 & 2.7 & 0 \\ 0 & 2.7 & 11.2969 & 1.8 \\ 0 & 0 & 1.8 & 5.8969 \end{bmatrix} kNm$$

Gk = 25 kN/m, and variable action, Qk = 10 kN/m, uniformly distributed along the beam.



$$G_k := 25 \frac{\text{kN}}{\text{m}} \qquad Q_k := 10 \frac{\text{kN}}{\text{m}}$$

$$W_{max} := \left(1.35 \cdot G_k + 1.5 \cdot Q_k\right) = 48.75 \frac{\text{kN}}{\text{m}}$$

$$W_{min} := \left(1.35 \cdot G_k\right) = 33.75 \frac{\text{kN}}{\text{m}}$$

Define Load Patterns for All 4 Cases Case I Case II Case III Case IV

$$FF := \begin{bmatrix} \begin{bmatrix} W_{max} \\ W_{min} \\ W_{max} \end{bmatrix} & \begin{bmatrix} W_{min} \\ W_{max} \\ W_{min} \end{bmatrix} & \begin{bmatrix} W_{max} \\ W_{max} \\ W_{min} \end{bmatrix} & \begin{bmatrix} W_{min} \\ W_{max} \\ W_{max} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 48.75 \\ 33.75 \\ 48.75 \end{bmatrix} & \begin{bmatrix} 33.75 \\ 48.75 \\ 33.75 \end{bmatrix} & \begin{bmatrix} 48.75 \\ 48.75 \\ 33.75 \end{bmatrix} & \begin{bmatrix} 33.75 \\ 48.75 \\ 48.75 \end{bmatrix} \end{bmatrix} \frac{kN}{m}$$

```
Call Program 7 to find support moments, upper & lower column moments
```

$FDN = 1$

$$M_{all} := NEW\_BMM\left(FF, K1, H_{UC}, H_{LC}, I_{beam}, I_{UC}, I_{LC}, FDN, L\right)$$

Upper Col Lower Col

$$M_{all} = \begin{bmatrix} \begin{bmatrix} -69.7 & 135.6 \end{bmatrix} & \begin{bmatrix} -93.9 & 93.9 \end{bmatrix} & \begin{bmatrix} -135.6 & 69.7 \end{bmatrix} & \begin{bmatrix} 37.2 \\ -22.2 \\ 22.2 \\ -37.2 \end{bmatrix} & \begin{bmatrix} 32.5 \\ -19.4 \\ 19.4 \\ -32.5 \end{bmatrix} \\ \begin{bmatrix} -45.3 & 106.7 \end{bmatrix} & \begin{bmatrix} -87.5 & 87.5 \end{bmatrix} & \begin{bmatrix} -106.7 & 45.3 \end{bmatrix} & \begin{bmatrix} 24.2 \\ -10.2 \\ 10.2 \\ -24.2 \end{bmatrix} & \begin{bmatrix} 21.1 \\ -8.9 \\ 8.9 \\ -21.1 \end{bmatrix} \\ \begin{bmatrix} -66.9 & 147.6 \end{bmatrix} & \begin{bmatrix} -115.1 & 79.7 \end{bmatrix} & \begin{bmatrix} -102.2 & 46.3 \end{bmatrix} & \begin{bmatrix} 35.7 \\ -17.4 \\ 12 \\ -24.7 \end{bmatrix} & \begin{bmatrix} 31.2 \\ -15.2 \\ 10.5 \\ -21.6 \end{bmatrix} \\ \begin{bmatrix} -46.3 & 102.2 \end{bmatrix} & \begin{bmatrix} -79.7 & 115.1 \end{bmatrix} & \begin{bmatrix} -147.6 & 66.9 \end{bmatrix} & \begin{bmatrix} 24.7 \\ -12 \\ 17.4 \\ -35.7 \end{bmatrix} & \begin{bmatrix} 21.6 \\ -10.5 \\ 15.2 \\ -31.2 \end{bmatrix} \end{bmatrix} kNm$$

Case I

Case II

Case III

Case IV

Beam1 Beam 2 Beam 3

Get Support Moments

L R L R L R

$$M_{sup} := M_{all\,\left[1..\text{rows}\left(M_{all}\right)\right]\left[1..\text{rows}(L)\right]} = \begin{bmatrix} \begin{bmatrix} -69.7 & 135.6 \end{bmatrix} & \begin{bmatrix} -93.9 & 93.9 \end{bmatrix} & \begin{bmatrix} -135.6 & 69.7 \end{bmatrix} \\ \begin{bmatrix} -45.3 & 106.7 \end{bmatrix} & \begin{bmatrix} -87.5 & 87.5 \end{bmatrix} & \begin{bmatrix} -106.7 & 45.3 \end{bmatrix} \\ \begin{bmatrix} -66.9 & 147.6 \end{bmatrix} & \begin{bmatrix} -115.1 & 79.7 \end{bmatrix} & \begin{bmatrix} -102.2 & 46.3 \end{bmatrix} \\ \begin{bmatrix} -46.3 & 102.2 \end{bmatrix} & \begin{bmatrix} -79.7 & 115.1 \end{bmatrix} & \begin{bmatrix} -147.6 & 66.9 \end{bmatrix} \end{bmatrix} kNm$$

Case I
Case II
Case III
Case IV

Get Column Moments

$$M_{col} := M_{all\,\left[1..\text{rows}\left(M_{all}\right)\right]\left[\left(\text{cols}\left(M_{all}\right)-1\right)..\text{cols}\left(M_{all}\right)\right]} = \begin{bmatrix} \begin{bmatrix} 37.2 \\ -22.2 \\ 22.2 \\ -37.2 \end{bmatrix} & \begin{bmatrix} 32.5 \\ -19.4 \\ 19.4 \\ -32.5 \end{bmatrix} \\ \begin{bmatrix} 24.2 \\ -10.2 \\ 10.2 \\ -24.2 \end{bmatrix} & \begin{bmatrix} 21.1 \\ -8.9 \\ 8.9 \\ -21.1 \end{bmatrix} \\ \begin{bmatrix} 35.7 \\ -17.4 \\ 12 \\ -24.7 \end{bmatrix} & \begin{bmatrix} 31.2 \\ -15.2 \\ 10.5 \\ -21.6 \end{bmatrix} \\ \begin{bmatrix} 24.7 \\ -12 \\ 17.4 \\ -35.7 \end{bmatrix} & \begin{bmatrix} 21.6 \\ -10.5 \\ 15.2 \\ -31.2 \end{bmatrix} \end{bmatrix} kNm$$

Case I

Case II

Case III

Case IV

Textbook Moment Distribution - 4 Iterations

Table 3.2 Moment distribution for the first loading case

	A			B				C				D	
	Cols. ($\sum M$)	AB		BA	Cols. ($\sum M$)	BC		CB	Cols. ($\sum M$)	CD		DC	Cols. ($\sum M$)
D.F.s	0.39	0.61		0.32	0.20	0.48		0.48	0.20	0.32		0.61	0.39
Load kN			292				135				292		
F.E.M.		+146		−146		+45.0		−45.0		+146		−146	
Bal.	−56.9	−89.1		+32.3	+20.2	+48.5		−48.5	−20.2	−32.3		+89.1	+56.9
C.O.		+16.2		−44.6		−24.2		+24.2		+44.6		−16.2	
Bal.	−6.3	−9.9		+22.0	+13.8	+33.0		−33.0	−13.8	−22.0		+9.9	+6.3
C.O.		+11.0		−5.0		−16.5		+16.5		+5.0		−11.0	
Bal.	−4.3	−6.7		+6.9	+4.3	+10.3		−10.3	−4.3	−6.9		+6.7	+4.3
C.O.		+3.4		3.4		−5.2		+5.2		+3.4		−3.4	
Bal.	−1.3	−2.1		+2.8	+1.7	+4.1		−4.1	−1.7	−2.8		+2.1	+1.3
M (kN m)	−68.8	+68.8		135.0	+40.0	+95.0		−95.0	−40.0	+135.0		−68.8	+68.8

CASE 1

Moment Distribution - EXCEL

37.16				**-22.20**				**22.20**				**-37.16**
0.01				-0.01				0.01				-0.01
0.01				-0.02				0.02				-0.01
0.04				-0.05				0.05				-0.04
0.10				-0.12				0.12				-0.10
0.28				-0.33				0.33				-0.28
0.71				-0.92				0.92				-0.71
2.28				-2.32				2.32				-2.28
3.35		Load		-7.46		Load		7.46		Load		-3.35
30.38	A	Max	B	-10.98		Min	C	10.98		Max	D	-30.38
0.21	**0.61**		**0.32**	**0.11**	**0.48**		**0.48**	**0.11**	**0.32**		0.61	**0.21**
0.18	**-146.25**		**146.25**	**0.09**	**-45.00**		**45.00**	**0.09**	**-146.25**		**146.25**	**0.18**
26.58	89.28		-32.27	-9.61	-48.40		48.40	9.61	32.27		**-89.28**	-26.58
	-16.13		44.64		24.20		-24.20		-44.64		**16.13**	
2.93	9.85		-21.94	-6.53	-32.91		32.91	6.53	21.94		**-9.85**	-2.93
	-10.97		4.92		16.45		-16.45		-4.92		**10.97**	
1.99	6.70		-6.81	-2.03	-10.22		10.22	2.03	6.81		**-6.70**	-1.99
	-3.41		3.35		5.11		-5.11		-3.35		**3.41**	
0.62	2.08		-2.70	-0.80	-4.04		4.04	0.80	2.70		**-2.08**	-0.62
	-1.35		1.04		2.02		-2.02		-1.04		**1.35**	
0.24	0.82		-0.98	-0.29	-1.46		1.46	0.29	0.98		**-0.82**	-0.24
	-0.49		0.41		0.73		-0.73		-0.41		**0.49**	
0.09	0.30		-0.36	-0.11	-0.55		0.55	0.11	0.36		**-0.30**	-0.09
	-0.18		0.15		0.27		-0.27		-0.15		**0.18**	
0.03	0.11		-0.13	-0.04	-0.20		0.20	0.04	0.13		**-0.11**	-0.03
	-0.07		0.06		0.10		-0.10		-0.06		**0.07**	
0.01	0.04		-0.05	-0.01	-0.07		0.07	0.01	0.05		**-0.04**	-0.01
	-0.02		0.02		0.04		-0.04		-0.02		**0.02**	
0.00	0.02		-0.02	-0.01	-0.03		0.03	0.01	0.02		**-0.02**	0.00
	-0.01		0.01		0.01		-0.01		-0.01		**0.01**	
0.00	0.01		-0.01	0.00	-0.01		0.01	0.00	0.01		**-0.01**	0.00
	0.00		0.00		0.01		-0.01		0.00		**0.00**	
0.00	0.00		0.00	0.00	0.00		0.00	0.00	0.00		**0.00**	0.00
32.52	**-69.68**		**135.59**	**-19.43**	**-93.95**		**93.95**	**19.43**	**-135.59**		**69.68**	**-32.52**