RAIN FALL ANALYSIS - Standardized Precipitation Index (SPI)

SAMPLE with First 10 Elements of Original DATA

$$X := \begin{bmatrix} 30.4 \\ 278.7 \\ 63.9 \\ 387.1 \\ 330 \\ 187.3 \\ 495.9 \\ 18 \\ 163.3 \\ 128.8 \end{bmatrix}$$

$\min(X) = 18$

$\max(X) = 495.9$

$n := \text{rows}(X) = 10$

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3. Dealing with zero values

Program 2: To deal with ln(0)

$n_{zero} := 0$

for $j \in [1..\text{rows}(X)]$

  if $X_j \le 0$

    $LNX_j := 0$

    $n_{zero} := n_{zero} + 1$

    $pos_{n_{zero}} := j$

  else

    $LNX_j := \ln(X_j)$

if $n_{zero} = 0$

  $pos := \text{"Undefined"}$

else

  $pos := pos$

ln(x) values

$$LNX = \begin{bmatrix} 3.414 \\ 5.63 \\ 4.157 \\ 5.959 \\ 5.799 \\ 5.233 \\ 6.206 \\ 2.89 \\ 5.096 \\ 4.858 \end{bmatrix}$$

$$X = \begin{bmatrix} 30.4 \\ 278.7 \\ 63.9 \\ 387.1 \\ 330 \\ 187.3 \\ 495.9 \\ 18 \\ 163.3 \\ 128.8 \end{bmatrix}$$

Index of the LNX array where LNX = 0

$pos = \text{"Undefined"}$

Clear j from memory $\text{Clear}(j) = 1$

$$X_{mean} := \frac{\sum X}{n} = 208.34$$

$$LNX_{mean} := \frac{\sum LNX}{n} = 4.924298109$$

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4. Gamma PDF and CDF

Number of zero values

$n_{zero} = 0$

Prob of zero values

$$Pr_{zero} := \frac{n_{zero}}{n} = 0$$

$A := \ln\left(X_{mean}\right) - LNX_{mean} = 0.414873251$

$\alpha := \frac{1}{4 \cdot A} \cdot \left(1 + \sqrt{1 + 4 \cdot \frac{A}{3}}\right) = 1.353582167$

$\beta := \frac{X_{mean}}{\alpha} = 153.91751239$

$\text{Gamma}(0.5) = 1.772456367$

$\text{Gamma}(\alpha) = 0.890795288$

$\text{Gamma}(1) = 1.000001604$

Eqn for Gamma function PDF

$$g(x) := \frac{1}{\beta^{\alpha} \cdot \text{Gamma}(\alpha)} \cdot x^{\alpha - 1} \cdot e^{-\frac{x}{\beta}}$$

Eqn. 1

$\text{Gamma}(\alpha) = 0.8908$ $\quad$ $\text{Gamma}(5) = 24$

$\text{Gamma}\left(\frac{1}{2}\right) = 1.7725$ $\quad$ $\sqrt{\pi} = 1.7725$

Eqn for Gamma function CDF

$$G(x\_val) := \frac{1}{\beta^{\alpha} \cdot \text{Gamma}(\alpha)} \cdot \int_0^{x\_val} x^{\alpha - 1} \cdot e^{-\frac{x}{\beta}} \, dx$$

Eqn. 2

$G(0) = 0$

$g(0) = 0$

PDF using Eqn 1

$$g1 := \overrightarrow{g(X)} = \begin{bmatrix} 0.0034 \\ 0.0015 \\ 0.0035 \\ 0.0008 \\ 0.0011 \\ 0.0023 \\ 0.0004 \\ 0.003 \\ 0.0026 \\ 0.003 \end{bmatrix}$$

$g\left(X_4\right) = 0.0008$

CDF using Eqn 2

$$G1 := \overrightarrow{G(X)} = \begin{bmatrix} 0.0923 \\ 1.8516 \\ 0.2522 \\ 2.8884 \\ 2.3286 \\ 1.0822 \\ 4.0407 \\ 0.0454 \\ 0.8983 \\ 0.6513 \end{bmatrix}$$

$G\left(X_4\right) = 2.8884$

$X_4 = 387.1$

Substitute g(x) in Eqn 2

$$Q(x\_val) := \int_0^{x\_val} g(x) \, dx$$

Eqn. 3

CDF using Eqn 3

$$G2 := \overrightarrow{Q(X)} = \begin{bmatrix} 0.0815 \\ 1.6353 \\ 0.2227 \\ 2.5509 \\ 2.0565 \\ 0.9557 \\ 3.5686 \\ 0.0401 \\ 0.7933 \\ 0.5752 \end{bmatrix}$$

$Q\left(X_4\right) = 2.5509$

$\text{appVersion}(3) = \text{"0.99.7251"}$

Both Eqns 2 & 3 are same, but the results of Integration (G1 & G2) are slightly different.
Note: Mathcad results are totally different from SS, but identical.