Integration using Runge-Kutta algorithms

$D(x, y) := y(x)$ $RK(ic, a, b) := al\_rkckadapt(ic, a, b, N, D)_{N+1\,2}$

$\varepsilon := 10^{-15}$ $N := 1000$ Set "Tools/Options/Calulation/Integrals: accuracy" to 10000.

$$y(x) := \frac{\log_{10}(x) \cdot x}{\operatorname{atan}(x) \cdot \left(1 - x^2\right)}$$

$RK(0, 0+\varepsilon, 1-\varepsilon) = -0.566035$

$$\int_{0+\varepsilon}^{1-\varepsilon} y(x)\,dx = -0.566194$$

$$\text{maple}\left(evalf\left(\int_0^1 eval\left(y(x), \operatorname{atan}(x) = arctan(x)\right) dx\right)\right) = -0.566034$$

$$y(x) := \frac{\operatorname{atan}(x) \cdot \log_{10}\left(1 - x^2\right) \cdot \log_{10}\left(1 + x^2\right)}{x^2}$$

$RK(0, 0+\varepsilon, 1-\varepsilon) = -0.058847$

$$\int_{0+\varepsilon}^{1-\varepsilon} y(x)\,dx = -0.058885$$

$$\text{maple}\left(convert\left(evalf\left(\int_0^1 eval\left(y(x), \operatorname{atan}(x) = arctan(x)\right) dx\right), rational\right)\right) = -0.058847$$

$$y(x) := \frac{\sin(x) \cdot \sec(x)}{1 + x \cdot \tan(x)} \cdot \log_{10}(x)$$

$RK(0, 0+\varepsilon, 1) = -0.094574$

$$\int_{0+\varepsilon}^{1} y(x)\,dx = -0.094574$$

$$\text{maple}\left(convert\left(evalf\left(\int_0^1 eval\left(y(x), \operatorname{atan}(x) = arctan(x)\right) dx\right), rational\right)\right) = -0.094574$$

$$y(x) := \frac{\operatorname{atan}(x) \cdot \log_{10}(x)}{x}$$

$RK(0, 0+\varepsilon, 1) = -0.420809$

$$\int_{0+\varepsilon}^{1} y(x)\,dx = -0.420968$$

$$\text{maple}\left(evalf\left(\int_0^1 eval\left(y(x), \operatorname{atan}(x) = arctan(x)\right) dx\right)\right) = -0.420808$$

$$y(x) := \frac{\log_{10}\left(1 - x^2\right) \cdot \operatorname{atan}(x)}{x}$$

$RK(0, 0+\varepsilon, 1-\varepsilon) = -0.22338$

$$\int_{0+\varepsilon}^{1-\varepsilon} y(x)\,dx = -0.223505$$

$$\text{maple}\left(evalf\left(\int_0^1 eval\left(y(x), \operatorname{atan}(x) = arctan(x)\right) dx\right)\right) = -0.22338$$

$$y(x) := \frac{\operatorname{atanh}(x)^2}{x^2}$$

$RK(0, 0+\varepsilon, 1-\varepsilon) = 1.644934$

$$\int_{0+\varepsilon}^{1-\varepsilon} y(x)\,dx = 1.644934$$

$$\int_{0+\varepsilon}^{1} \ldots$$

$$\text{maple}\left(evalf\left(\int_0^1 eval\left(y(x), \text{atanh}(x) = arctanh(x)\right) dx\right)\right) = 4.207199 - 3.875785\cdot i$$

Maple fails maybe because the symbolic expression is so complicate, that it's numerical evaluation is so difficult at the point that it is useless, or it have a bug for this case:

$$I := \text{maple}\left(\int_0^1 eval\left(y(x), \text{atanh}(x) = arctanh(x)\right) dx\right)$$

$$I = -\frac{Sum\left(\frac{4\cdot(-1)^{-k1}\cdot\left(2\cdot\left(-\Psi\left(\frac{3+2\cdot\_k1}{2}\right)+\Psi\left(\frac{1+2\cdot\_k1}{2}\right)\right)-\pi\cdot(i+\tan(\pi\cdot\_k1))\right)\cdot\cos(\pi\cdot\_k1)}{(1+2\cdot\_k1)^2}, \_k1 = 0_{.infinity} - \pi^2\right)}{4}$$

New versions of maple have options for expecify the numerical procedure for evaluate the definite integral: https://www.maplesoft.com/support/help/Maple/view.aspx?path=evalf/Int

Double integrals

$$\varepsilon := 10^{-7}$$

$$DInt\left(f(2), x_1, x_2, y_1, y_2, n_x, n_y\right) := \left| \begin{array}{l} RK_1(a, b, n) := \text{al\_rkckadapt}([0], a, b, n, D(x, y))_{n+1\,2} \\ D(y, u) := \left| \begin{array}{l} D(x, v) := [f(x, y)] \\ RK_1(x_1, x_2, n_x) \end{array} \right. \\ RK_1(y_1, y_2, n_y) \end{array} \right.$$

$$f(x, y) := \frac{1 - x}{1 - x^2\cdot y^2}$$

$$DInt(f(x, y), 0, 1, 0, 1-\varepsilon, 20, 20) = 0.540553$$

$$\text{maple}\left(evalf\left(\int_0^1\int_0^1 f(x, y)\, dx\, dy\right)\right) = 0.540553$$

$$f(x, y) := \frac{1}{(x+y)\cdot\left(1 - x^2\cdot y^2\right)}$$

$$DInt(f(x, y), 0.5, 1, 0.5, 1-\varepsilon, 20, 20) = 0.290638$$

$$\text{maple}\left(\text{Re}\left(evalf\left(\int_{0.5}^1\int_{0.5}^1 f(x, y)\, dx\, dy\right)\right)\right) = 0.290638$$

References

Maritn's post about Maxima quadrature:
- https://en.smath.com/forum/yaf_poststst25506_Smath-cananot-solve-this-integral-No5-while-Mathcad-Prin

Single int: A bug in the odesolvers and its solution:
- https://en.smath.com/forum/yaf_postsm56081_Ode-solvers-implementation.aspx
- https://en.smath.com/forum/yaf_postsm57304_ODE-Solvers.aspx#post57304

Double int:
- https://en.smath.com/forum/yaf_postsm61577_Double-Integration-using-ode-solvers.aspx
- https://en.smath.com/forum/yaf_postsm85273_Mathcad-Toolbox.aspx#post85273

Maple convert/rational:
- https://en.smath.com/forum/yaf_postsm82867_Maple-Tools.aspx#post82867

Alvaro

$$\text{appVersion}(4) = \text{"1.73.9126.0"}$$