ElasticCatenary

Notation from https://en.wikipedia.org/wiki/Catenary#Elastic_catenary

$\text{Clear}(\alpha, \beta, wo, To, E, p_A, p_B, p_O, p, a, xo, yo) = 1$

$T := \sqrt{p^2 \cdot wo^2 + To^2}$

Numeric Elastic Catenary

$$\begin{cases} x'(p) = \frac{To}{E} + \frac{wo}{T} \\ y'(p) = wo \cdot p \cdot \frac{T + E}{T \cdot E} \\ x(p_A) = x_A \quad y(p_A) = y_A \end{cases}$$

$$RK(p_A, p_B) := \text{Rkadapt}\left(\begin{Bmatrix} x(p) \\ y(p) \end{Bmatrix}, p_B, N\right)$$

$$ECat(vo) := \begin{vmatrix} eq(p) := \begin{vmatrix} RK := RK(p_1, p_2) \\ \begin{bmatrix} RK_{\text{rows}(RK)\,2} - x_B \\ RK_{\text{rows}(RK)\,3} - y_B \end{bmatrix} \end{vmatrix} \\ p := \text{al\_nleqsolve}(vo^T, eq) \\ \text{augment}(p^T, [RK(p_1, p_2)]) \end{vmatrix}$$

Symbolic Elastic Catenary

$$\begin{cases} x_s(p) := \text{asinh}\left(\frac{p \cdot wo}{To}\right) + \frac{To}{E} \cdot p + \alpha \\ y_s(p) := \frac{T}{wo} + \frac{wo}{2 \cdot E} \cdot p^2 + \beta \end{cases}$$

Catenary by 3 points

$$f(x) := a \cdot \cosh\left(\frac{x - xo}{a}\right) + yo$$

$$Cat(vo) := \left| \text{FindRoot}\left(\begin{cases} f(x_A) = y_A \\ f(x_B) = y_B \\ f(x_s(p_O)) = y_s(p_O) \end{cases}, vo^T\right)^T \right.$$

Plot

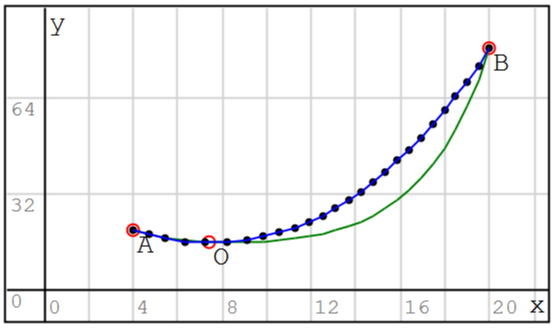
$$Plot(RK, vo) := \begin{vmatrix} [a\ xo\ yo] := Cat(vo) \\ [P := \text{col}(RK, 1)\ \ X := \text{col}(RK, 2)\ \ Y := \text{col}(RK, 3)] \\ ABO := \text{stack}([x_A\ y_A], [x_B\ y_B], [x_s(p_O)\ y_s(p_O)]) \\ \begin{cases} \text{augment}(\overrightarrow{x_s(P)}, \overrightarrow{y_s(P)}) \\ \text{augment}(X, Y, ".") \\ \text{augment}(X, \overrightarrow{f(X)}) \\ \text{augment}(ABO, "o", 10, "red") \\ \text{augment}(ABO, \text{stack}("A", "B", "O"), 10) \end{cases} \end{vmatrix}$$

$Params := \text{stack}("wo", "To", "E")$

$N := 25$

Example $wo := 15$ $To := 20$ $E := 30$ $[x_A\ y_A] := [4\ 20]$ $[x_B\ y_B] := [20\ 80]$

$$\begin{vmatrix} [p_A\ p_B\ RK] := ECat([-1\ 1]) \\ p_O := \text{eval}(p_A \cdot (p_A > 0)) \\ \alpha := \text{roots}(x_s(p_A) = x_A, \alpha) \\ \beta := \text{roots}(y_s(p_A) = y_A, \beta) \end{vmatrix}$$



$Plot(RK, [1\ 1\ -1])$

Results $[p_O\ p_A\ p_B] = [0\ -2.87\ 14.27]$

$[\alpha\ \beta] = [7.42\ 14.78]$

Minimun $[p_O\ x_s(p_O)\ y_s(p_O)] = [0\ 7.42\ 16.11]$

If O is between A and B, then p.O = 0.

$\text{findrows}(RK, \min(\text{col}(RK, 3)), 3) = [-0.13\ 7.24\ 16.12]$

$\begin{vmatrix} X := \text{col}(RK, 2) \\ Y := \text{col}(RK, 3) \end{vmatrix}$

Arc length

$$\int_{p_A}^{p_B} \sqrt{\left(\frac{d}{dp} x_s(p)\right)^2 + \left(\frac{d}{dp} y_s(p)\right)^2}\, dp = 71.7$$

$$\sum_{k=2}^{\text{rows}(RK)} \text{norme}\left(\begin{bmatrix} X_k - X_{k-1} \\ Y_k - Y_{k-1} \end{bmatrix}\right) = 71.66$$

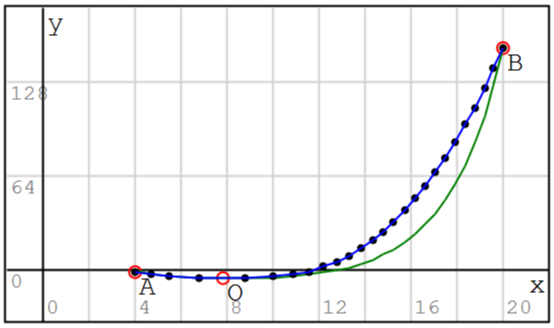
Example $wo := 30$ $To := 5$ $E := 4$ $[x_A\ y_A] := [4\ 0]$ $[x_B\ y_B] := [20\ 150]$ $\text{Clear}(\alpha, \beta) = 1$

$\begin{bmatrix} p_A & p_B & RK \end{bmatrix} := ECat\left(\begin{bmatrix} -1 & 1 \end{bmatrix}\right)$

$p_O := \text{eval}\left(p_A \cdot \left(p_A > 0\right)\right)$

$\alpha := \text{roots}\left(x_s\left(p_A\right) = x_A, \alpha\right)$

$\beta := \text{roots}\left(y_s\left(p_A\right) = y_A, \beta\right)$



Plot$(RK, [1\ 1\ -1])$

$\text{Clear}(\alpha, \beta) = 1$

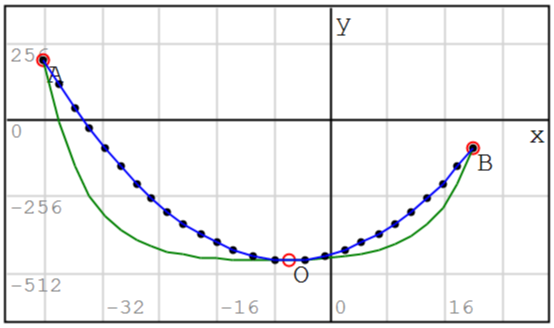
Example $wo := 3$ $To := 10$ $E := 40$ $\begin{bmatrix} x_A & y_A \end{bmatrix} := \begin{bmatrix} -40 & 200 \end{bmatrix}$ $\begin{bmatrix} x_B & y_B \end{bmatrix} := \begin{bmatrix} 20 & -90 \end{bmatrix}$

$\begin{bmatrix} p_A & p_B & RK \end{bmatrix} := ECat\left(\begin{bmatrix} -1 & 1 \end{bmatrix}\right)$

$p_O := \text{eval}\left(p_A \cdot \left(p_A > 0\right)\right)$

$\alpha := \text{roots}\left(x_s\left(p_A\right) = x_A, \alpha\right)$

$\beta := \text{roots}\left(y_s\left(p_A\right) = y_A, \beta\right)$



Plot$(RK, [1\ 1\ -1])$

Example Guess for p.A, p.B Guess for a, xo, yo Point A RK Intervals Plot margin

$vo_1 := \begin{bmatrix} -40 & 10 \end{bmatrix}$ $vo_2 := \begin{bmatrix} 10 & -10 & -10 \end{bmatrix}$ $\begin{bmatrix} x_A & y_A \end{bmatrix} := \begin{bmatrix} 0 & 0 \end{bmatrix}$ $N := 25$ $\delta\pi := 0.20$

Weight per unit length Tension Stiffness per length

$\text{Clear}(\alpha, \beta) = 1$

$wo := 2 \cdot \kappa_1 = 4$ $To := 20 \cdot \kappa_2 = 44$ $E := 40 \cdot \kappa_3 = 148$

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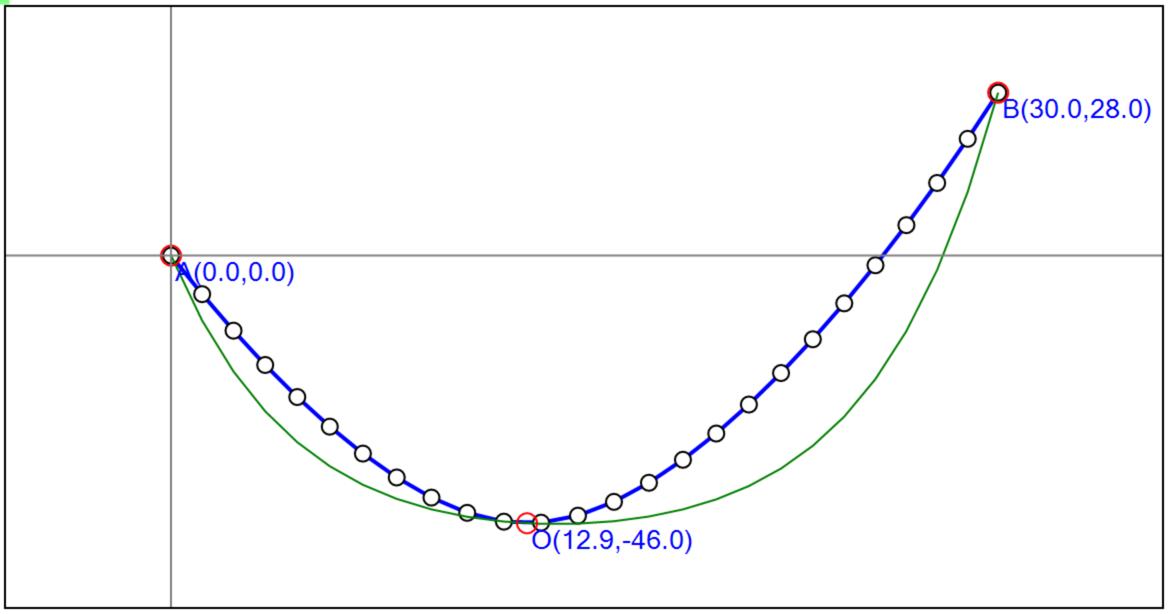

$$\int_{p_A}^{p_B} \sqrt{\left(\frac{d}{d\,p} x_s(p)\right)^2 + \left(\frac{d}{d\,p} y_s(p)\right)^2}\, d\,p = 125.46$$

$$\begin{bmatrix} p_A \\ p_B \\ p_O \end{bmatrix} = \begin{bmatrix} -36.95 \\ 50.01 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_s\left(p_O\right) \\ y_s\left(p_O\right) \end{bmatrix} = \begin{bmatrix} 12.91 \\ -46.01 \end{bmatrix}$$

Alvaro $\text{appVersion}(4) = $ "1.2.9018.0"